Real Business Cycles in a Commodity-Exporting Economy∗

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Abstract

This paper analyzes the business cycle of a small commodity-exporting economy. Using Chilean data, we show SVAR evidence of the effect of a commodity price shock on the non-commodity economy. A positive shock to the commodity price leads to an increase in both non-commodity output and employment, a real exchange rate appreciation, an increase in investment and a decrease in the non-commodity trade balance. Moreover, we find that commodity price fluctuations contribute to 40 percent of the variance of non-commodity output. We build a theoretical three-sector model where the commodity good is produced with a technology that requires non-tradable investment and intermediate goods. An increase in the price of commodity rises the demand and the price of non-tradable goods. It also induces a factor reallocation and an increase in total labor demand, leading to an expansion of the non-commodity economy. Our quantitative results have a good fit to the data and emulate reasonably well the dynamic response of the non-commodity economy to a commodity price shock.

JEL Classification: E32, F41, F44.
Keywords: Terms of trade, real exchange rate, business cycles, small open economy.

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1 Introduction

Modern real business cycle theory was developed to explain the source of fluctuations in aggregate activity. In closed economy models most shocks are unobservable and therefore strong identification assumptions are needed to uncover their importance. Small open economies offer an ideal laboratory to gauge the nature of fluctuations as many shocks are both observable and exogenous. Ever since the seminal work of Mendoza (1991), researchers have devoted efforts to link foreign factors to the aggregate fluctuations in small open economies. Terms of trade have been singled out as an important shock shaping business cycles, specially in economies that depend heavily on commodity production. For example, Mendoza (1995) found that the terms of trade were a major driver of short-run fluctuations. Recently, Schmitt-Grohé and Uribe (2015) documented a disconnect between empirical evidence for a panel of countries that shows a small effect of the terms of trade on aggregate activity and calibrated real business cycle models that assign a large role to relative price fluctuations.

In this paper we study the business cycle of a commodity exporter. In particular, we study the Chilean economy where the commodity sector is relatively isolated from the rest of the economy and yet is commonly believed to influence it. Recently, Chile has pursued an active counter-cyclical fiscal rule. That is, in times of commodity price boom the government reduces government expenditure below its long run trend. In the traditional Dutch disease type of argument, a positive windfall generates a boom, and a contraction in the tradable sector. However, with a countercyclical fiscal policy the effects are the opposite.

We document that copper prices have a significant impact on the non-commodity economy. We use a SVAR analysis for Chile to show that a shock to the price of copper leads to an increase in non-copper GDP and employment, an increase in investment, a real exchange rate appreciation and a decrease in the non-commodity trade balance to GDP ratio. Moreover, our variance decomposition analysis shows that the price of copper explains 40 percent of the variance of non-copper GDP. To explain these facts we develop a quantitative three-sector real business model where sectoral linkages from the commodity sectors are a key force. We calibrate the model using several steady-state and dynamic moments to reflect realistic features of the Chilean economy. The model has a good fit to several unconditional moments. Moreover, the impulse response function of several key variables are in line with the empirical SVAR analysis. Finally, we show that omitting any of these factors leads to a failure of the model to capture the dynamic response observed in the SVAR.
We propose a simple model to rationalize the co-movement of the commodity sector and the non-commodity economy in the presence of a countercyclical fiscal rule. In our theoretical model investment in the commodity sector requires non-tradable goods. Importantly, the investment dynamics features adjustment costs which is a key mechanism to match the dynamic response of activity to a copper price shock. The commodity sector demands intermediate goods that are in part non-tradable. Labor and capital are mobile across the tradable and non-tradable sectors\(^1\) whereas commodity investment is sector specific and cannot be reallocated. Finally, consistent with the evidence we include a government that runs a countercyclical fiscal rule, so government transfers to households decline when commodity and non-commodity GDP are below trend.

The transmission mechanism works thought the relative price of non-tradables and the role of reallocation. An increase in the commodity price increases the demand for both commodity investment and intermediate goods used in the commodity production. Given the fact that commodity investment and a proportion of intermediate goods are non tradable, the demand for this type of goods increases, generating a real exchange rate appreciation (a rise in the price of non-tradables). The change in the real exchange rate increases the value of the marginal productivity of non-tradable labor and capital. This induces a reallocation of labor and capital from the tradable to the non-tradable sector. In addition, because the non-tradable sector is labor intensive, an increase in the real exchange rate shifts up the labor demand schedule which leads to an increase in real wages, and an overall increase in total labor in the non-commodity sector.\(^2\) Since capital reacts slowly, due to the presence of adjustment cost, and labor increases on impact, non-tradable GDP increases, determining an expansion of non-commodity GDP.

Our paper contributes to the literature that examines the role of terms of trade shocks in small open economies. Schmitt-Grohé and Uribe (2015) based on structural vector autoregression models suggests that world shocks transmitted by the terms of trade alone explain on average only 10 percent of the variation in output and other indicators of aggregate activity in poor and emerging countries. However, Fernández et al. (2016), suggest that the terms of trade and other single measures of world prices may provide insufficient information to uncover the channels through which world shocks are transmitted to domestic economies. It is in this context that the use of more disaggregated world price measures could better account for world shocks.

\(^1\)As is well documented in the literature (see Schmitt-Grohe and Uribe 2016), we assume that the non-tradable sector is more labor intensive than the tradable sector.

\(^2\)We assume that the production of non-traded goods is more labor intensive and, as a consequence, an expansion of this sector requires not only the labor that it is reallocated from the traded sector, but also additional labor.
Fernández Martin et al. (2015), conclude that country-specific commodity price measures explain about 50 percent of aggregate fluctuations in Brazil, Chile, Colombia, and Peru over the period 2000 to 2014. Similar evidence is provided by Shousha (2015), who documents that in a group of advanced and emerging commodity exporters world price shocks played a major role in driving short-run fluctuations since the mid 1990s.

This paper is also related to the Dutch disease literature (Corden and Neary (1982), Pieschacón (2012), Céspedes and Velasco (2014) and others). In those papers an increase in commodity revenue leads to windfall from government spending which increases the relative price of non-traded goods and a loss in competitiveness in the tradable sector (usually manufacturing). In our paper we emphasize that the same Dutch disease phenomena could occur in countries running a countercyclical fiscal policy. Our mechanism indeed emphasizes the role of sectoral linkages, instead of government spending, as an explanation of the real exchange rate appreciation and the expansion of the non-commodity GDP and employment observed in the data.

The paper is divided into five sections. First, we provide empirical evidence of the dynamic impact on activity to a copper price shock and the counter-cyclicality of the fiscal policy in Chile. Second, we present a simple theoretical model that highlights the role of linkages between the commodity sector and the rest of the economy. Third, we present the benchmark calibration, show several fitness measures and compare the impulse responses with the SVAR analysis. Fourth, we analyze and compare the performance of alternative models. Finally, we conclude.

2 Stylized Facts

In this section we conduct a standard SVAR analysis and show how a copper price shock propagates to the rest of the economy. Our main mechanism in the model is that investment and a fraction of intermediate goods used to produce commodity require non-tradable goods. We present evidence linking the investment and intermediate goods in the copper sector with demand for non-traded goods (services and construction). Finally, we show that net transfers from the government are countercyclical.

2.1 The Importance of Copper Price Shocks: SVAR Evidence

Here we use SVAR analysis to document a significant effect of copper price innovations on aggregate fluctuations in Chile. We proceed as in Schmitt-Grohé and Uribe (2015) and consider
the following SVAR specification

\[ Z_t = A Z_{t-1} + u_t \]

where we define the vector of observable \( Z_t = [p_{co}^t, v_y^t, l_t, y_{nc}^t, i_t, rer_t, tby_t] \). The variables \( p_{co}^t, l_t, y_{nc}^t, i_t, rer_t \) are log detrended real price of copper, non-commodity labor and GDP, real gross investment and the real exchange rate. The variables \( v_y^t \) and \( tby_t \) are detrended total governmen transfers to non-commodity GDP ratio, and trade balance to non-commodity GDP.

We assume that the off-diagonal elements of the first row of the matrix A are zero and use Cholesky decomposition of the matrix of shocks. The SVAR is estimated using quarterly data from 1996.Q1 to 2013.Q4. Figure (1) displays the response to a one standard deviation shock in the real price of copper. The bold line corresponds to the point estimates. The dotted lines indicate 95 percent confidence intervals. The results are consistent with a countercyclical fiscal policy, that is net transfers to GDP fall gradually in response to an increase in the commodity price. We also observe statistically significant responses of aggregate investment and non-commodity GDP. The relative price of non-tradable goods increases upon impact (a real exchange rate appreciation) to return gradually to its steady-state value. Finally, the trade balance deteriorates rapidly few quarters after the shock.
We also compute the share of the variance of non-commodity GDP explained by the price of copper, and conclude that it is 40 percent. This is a large number compared with those reported in Schmitt-Grohé and Uribe (2015) which report an average of 10 percent.\footnote{It is worth mentioning that our analysis differs from Schmitt-Grohé and Uribe (2015). Their estimates report the variance decomposition of aggregate GDP to a term of trade shock. In contrast, we report the variance explained in non-commodity GDP to a commodity price shock.}

### 2.2 Evidence of Strong Inter-sectoral Linkages

There is evidence of sectoral linkages between the commodity sector and the rest of the economy. In Table 1 we document several characteristics of the copper industry in Chile. The copper
industry is capital intensive and uses only 5 percent of the economy-wide labor force. More importantly, the share of intermediate goods demanded from the non-commodity economy in mining production is 21 percent. A large part of mining investment is in non-tradable goods (exploration and services) produced in the non-commodity economy. Sixty-six percent of mining investment is composed of exploration expenses, materials and construction. The rest is labeled as manufacturing, which can be either produced locally or imported. Therefore, we interpret the 66 percent as a lower bound value. At the business cycle frequency we find an unconditional correlation of 0.55 between real copper prices and the non-commodity GDP. Using annual data we find that total investment and the price of copper are mildly positively correlated, with a correlation of 0.17.

Table 1: Sectoral Linkages Between the Commodity Sector and the Non-Commodity Economy.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input-output linkages (I-O Matrix 2013)</td>
<td></td>
</tr>
<tr>
<td>Share of intermediate goods in copper production</td>
<td>0.2</td>
</tr>
<tr>
<td>Share of labor in mining</td>
<td>0.07</td>
</tr>
<tr>
<td>Surplus (including rents)</td>
<td>0.5</td>
</tr>
<tr>
<td>Steady state (Average 1996-2013)</td>
<td></td>
</tr>
<tr>
<td>Share of exploration and construction investment over total commodity investment</td>
<td>0.66</td>
</tr>
<tr>
<td>Correlation of copper price with non-commodity GDP</td>
<td>0.55</td>
</tr>
<tr>
<td>Correlation of copper price with total investment</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Notes: For input-output linkages, we use the 2013 input-output matrix for Chile. The share of intermediate goods in copper excludes intermediates from mining and energy. The share of exploration and construction over total mining investment is the average for 1996-2013, using annual investment data. The correlation of detrended noncommodity GDP and detrended investment with the real copper price (deflated by US inflation) is computed using quarterly data from 1996.Q1 to 2013.Q4. Source: Central Bank of Chile (www.bcentral.cl).

2.3 Fiscal Rule

This section estimates the fiscal rule using data on net government transfers, output and commodity revenues, in order to complement the result of the SVAR analysis and document the

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4Unfortunately, we do not have access to quarterly series for mining investment.
nature of the fiscal policy in Chile. We will use the fiscal rule and its estimates in the real business cycle model we calibrate below. Following Fernández-Villaverde et al. (2015) we postulate that net transfers from the government to households, $v_y$, is described by the following specification:

$$v_y = \rho v_y - 1 + \theta_o \tilde{r}_o - 1 + \theta_y \tilde{y}_M - 1 + \epsilon_t$$

(2.1)

where $v_y$ is the quadratic detrended net transfer over non-commodity GDP ratio. The variable $\tilde{r}_o - 1$ represents the detrended log deviation of commodity income that the country receives. To construct this variable, nominal commodity income is deflated by a measure of tradable prices, so changes in the price of the commodity are captured by $\tilde{r}_o - 1$. The variable $\tilde{y}_M - 1$ is lagged detrended log real non-commodity GDP and we assume the error term follows a normal distribution $N(0, \sigma_v^2)$. The above fiscal policy rule allows for two feedback responses, one from the state of the business cycle in the non-commodity sector, captured by $\theta_y$ and another from the fiscal response to commodity windfalls, $\theta_o$.

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5Net transfers are constructed as the difference between total primary government spending and total government income, excluding income related to the commodity. Non-commodity GDP is constructed as total GDP excluding mining GDP. The source of data is the Central Bank of Chile and the Government Budget Office (Dirección de Presupuestos).

6Nominal commodity income is total copper nominal GDP obtained from the Central Bank of Chile. The measure of tradable prices is constructed as the product of the US CPI inflation index and the nominal exchange rate.

7To construct constant price non-commodity GDP, we exclude mining GDP from total GDP. The source of this data is the Central Bank of Chile.
Table 2: Fiscal Rule Regressions

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged v/y</td>
<td>ρ</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.409***</td>
<td>0.752***</td>
<td>0.330***</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.198)</td>
<td>(0.121)</td>
</tr>
<tr>
<td>Non-Commodity GDP</td>
<td>θ_y</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.186*</td>
<td>-0.031</td>
<td>-0.374*</td>
</tr>
<tr>
<td></td>
<td>(0.110)</td>
<td>(0.079)</td>
<td>(0.194)</td>
</tr>
<tr>
<td>Commodity income</td>
<td>θ_o</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.042***</td>
<td>-0.023</td>
<td>-0.039***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.014)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Adjusted R²</td>
<td></td>
<td>0.64</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Notes: This regression is performed using quarterly data from 1996.Q1 to 2013.Q4. The variable \( v/y \) is fiscal expenditures minus fiscal income (excluding copper) over non-copper GDP. Non-commodity GDP is measured in constant prices, whereas commodity GDP is deflated by a measure of tradable prices (US CPI times the nominal exchange rate). National Accounts series are taken from the Central Bank of Chile.

Table 2 reports the regression results from estimating equation (2.1). We estimate the fiscal rule equation from 1996.Q1 to 2013.Q4 and find that, in terms of the non-commodity business cycle, there is a countercyclical fiscal policy response. In particular, \( θ_y \) is negative and statistically different from zero. This means that in economic downturns the fiscal authority expands transfers to households and vice versa in economic expansions. The fiscal policy response to commodity windfalls is also countercyclical: \( θ_o \) is negative and statistically different from zero. This means that when commodity income increases above its trend, the fiscal authority increases its savings and vice versa. Now, the whole sample results are determined by the behavior of the fiscal authority since the early 2000s. In particular, in the subsample that goes from 1999.Q4 to 2013.Q4 the response to both the non-commodity GDP cycle and commodity income are larger (in absolute value) and statistically different from zero when compared to the previous subsample, 1996.Q1 to 1999.Q4. In terms of magnitude, since 2000, transfers as a percentage of non-commodity GDP increase by 0.37% in the short run, when real non-commodity GDP declines by 1%. A decline of commodity revenues of 1% generates, in the short run, an increase

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8 Chile adopted in 2000 a structural surplus rule as the basis for the fiscal policy from 2001 onwards. Under this rule, the government is committing itself to maintain its expenditures equal to the structural revenue minus the target for the structural surplus, which is expressed in terms of the GDP. The calculation of the structural balance depends on trend output, the long-run copper price and the structural income from Codelco’s molybdenum sale.

9 In the long run, the increase in transfers is 0.5%. This value is obtained by dividing the estimated \( θ_y \) value
in net transfers as a percentage of non-commodity GDP of 0.039%. In the long run, this response is -0.058%.

In what follows we will use the estimates from the latter subsample as a starting point to calibrate the relevant fiscal moments in our model.

3 The Model

Motivated by the empirical evidence we construct a quantitative three-sector real business cycle model. The aim is to explain several unconditional and conditional empirical moments as well as the dynamic response of key variables to a commodity price shock. The economy is divided in two; a non-commodity economy and a relatively isolated commodity sector. In turn, the non-commodity economy consists of two sectors producing tradable and non-tradable goods.

3.1 Consumers

In this economy, households own firms in the traded and non-traded sectors. Households also supply labor and rent capital to the different industries. The government and foreign investors extract the commodity using capital and intermediate goods produced by firms in the non-commodity sector. Households receive the value of output from the traded and non-traded sectors and also receive payments from commodity producers for inputs they provide. Finally, the government can transfer resources or tax households according to a fiscal policy rule.

There is a large number of households with the following preferences:

\[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( c_t - \frac{l_t^c}{\omega} \right)^{1-\sigma} - 1 \right] \]

where \( l_t \) is labor, and \( c_t \) is an aggregate composite of tradable and non-tradable goods given by:

\[ c_t = c_{N,t}^{\chi} c_{T,t}^{1-\chi} \tag{3.1} \]

where \( \chi \) is the elasticity of substitution between tradable and non-tradable consumption goods. by \((1-\rho)\).
Households maximize utility subject to the following sequential budget constraint:

\[
\begin{align*}
    d_t + y_{T,t} + v_t + p_{N,t}y_{N,t} + p_{O,t}y_{O,t} - \pi_{O,t} + (1 + r_{t-1})d_{t-1} + c_{T,t} + p_{N,t}c_{N,t} \\
    + i_{M,t} + p_{N,t}i_{O,t} + p_{t}^{o,m}y_{t}^{o,m}
\end{align*}
\]  

(3.2)

where \(y_{T,t}, y_{N,t}\) and \(y_{O,t}\) denote the production of tradable, non-tradable and commodity sold at their respective prices \(p_{i,t}\) where \(i = T, NT, O\). For simplicity we choose the price of tradable goods as the numeraire, so \(p_T = 1\).

The right-hand side of equation (3.2) represents the flow of households expenditures, where \(c_{T,t}\) and \(p_{N,t}c_{N,t}\) denote the consumption of tradable and non tradable goods. The household invests in non-commodity capital, \(i_{M,t}\), which is tradable, and in commodity-specific capital, \(i_{O,t}\), which is non tradable. Households demand intermediate goods, \(p_{t}^{o,m}y_{t}^{o,m}\), which are sold to commodity producer firms and pay interest on the level of debt of the last period, \((1 + r_{t-1})d_{t-1}\). The left-hand side of equation (3.2) represents the flow of income to households, which receive the value of tradable and non tradable output. In addition, they receive transfers from the government if \(v_t\) is positive, or are taxed if \(v_t\) is negative. Finally, they are paid for renting capital and selling intermediate goods to the commodity sector. Those payments are equal to \(p_{O,t}y_{O,t} - \pi_{O,t}\), where \(\pi_{O,t}\) are the rents of the mining sector.\(^{10}\)

Following Christiano et al. (2005), we assume that the capital accumulation equations for non-commodity and commodity sectors are given by:

\[
\begin{align*}
    k_{M,t+1} = (1 - \delta_M)k_{M,t} + i_{M,t} \left[ 1 - \frac{\phi_M}{2} \left( \frac{i_{M,t}}{i_{M,t-1}} - 1 \right)^2 \right] \\
    k_{O,t+1} = (1 - \delta_O)k_{O,t} + i_{O,t} \left[ 1 - \frac{\phi_O}{2} \left( \frac{i_{O,t}}{i_{O,t-1}} - 1 \right)^2 \right]
\end{align*}
\]  

(3.3) (3.4)

The adjustment cost is a function of the rate of change in investment, rather than the investment-capital ratio as in the standard adjustment cost function (Hayashi (1982)).

Households choose labor \(l_t\), investment and capital in each sector, tradable consumption, \(c_{T,t}\), non-tradable consumption \(c_{N,t}\), and next period’s debt level \(d_{t+1}\) to maximize utility subject to the sequential budget constraint and the capital accumulation equations (3.3)-(3.4).

From the first-order conditions of households’ maximization problem it is possible to derive

\(^{10}\) As a result factor payments are given by \(p_{O,t}y_{O,t} - \pi_{O,t} = u_{O,t}k_{O,t} + p_{t}^{o,m}y_{t}^{o,m}\).
the relative demand for non tradable consumption and the households’ labor supply:

\[
\frac{c_{N,t}}{c_{T,t}} = \left(\frac{1 - \chi}{\chi}\right) \frac{1}{p_{N,t}} \tag{3.5}
\]

\[
l_t = Z \left( \frac{w_t}{p_{N,t}} \right)^{\frac{1}{1-\chi}} \tag{3.6}
\]

where \( Z = \left[ \chi \chi(1 - \chi)^{1-\chi} \right]^{\frac{1}{1-\chi}} \). The relative demand for consumption goods is inversely related to the relative price of traded to non-traded goods, \( 1/p_{N,t} \). The labor supply is increasing in wages (relative to the price of the consumption basket, \( p_{C,t} = p_{N,t}^{1-\chi} \)) and is independent from the income level.

### 3.2 Production

**Commodity goods.** Commodity goods are produced using capital \( k_{O,t} \), and intermediate inputs from non-commodity sector, \( y_{om} \). To reflect resource scarcity, we assume commodity goods are produced using the following decreasing return to scale technology

\[
y_{O,t} = A_{O,t}(k_{O,t})^{\alpha_O}(y_{om})^{\beta_O} \tag{3.7}
\]

where \( A_{O,t} \) is a commodity-specific technological parameter. Producers maximize profits by choosing capital and intermediate inputs. The rental rate for commodity capital, \( u_{O,t} \), and the price for intermediate goods, \( p_{om} \), are taken as given. Profits in this sector are expressed as

\[
\pi_{O,t} = p_{O,t} y_{O,t} - u_{O,t} k_{O,t} - p_{om} y_{om}
\]

**Non-commodity Firms.** Tradable and non-tradable goods are produced with capital and labor. Tradable production has constant returns to scale with technologies

\[
y_{T,t} = A_{T,t} k_{T,t}^{\psi} l_{T,t}^{1-\psi} \tag{3.8}
\]

where \( A_T \) denotes the technological parameter, \( k_T \) is capital, and \( l_T \) is labor input. Firms decide the input bundle that maximizes profits, \( \pi_{T,t} = y_{T,t} - u_{M,t} k_{T,t} - w_t l_{T,t} \), taking the rental price of capital, \( u_{M,t} \), and wages, \( w_t \), as given.
Non-tradable output is produced using a decreasing return to scale technology

\[ y_{N,t} = A_N l_{N,t}^{\alpha_N} N_{N,t}^{\beta_N} \]  

(3.9)

where \( A_N \) denotes the technological parameter. Firms maximize profits, \( \pi_{N,t} = p_{N,t} y_{N,t} - u_{M,t} k_{N,t} - w_t l_{N,t} \), by choosing \( k_N \) and \( l_N \). We assume perfect input mobility across sectors, that is, \( l_t = l_{T,t} + l_{N,t} \) and \( k_t = k_{T,t} + k_{N,t} \).

Intermediate goods are produced using tradable and non tradable goods, which are combined according to the following Leontief production function

\[ y_{om,t} = \min \left\{ \frac{y_{om,t}}{a}, \frac{y_{om,t}}{b} \right\} \]

where \( a \) and \( b \) are time-invariant parameters reflecting the relative importance of traded and non-traded goods in the production of intermediates. It can be shown that the price of intermediate goods is given by \( p_{om,t} = a + b p_{N,t} \).

Now, from the first-order condition of firms, we can obtain the relative demands for inputs. Given that capital and labor are mobile across sectors, tradable and non-tradable firms face the same wage, \( w_t \), and capital rental rate, \( u_{M,t} \). As a consequence, the relative demand for inputs in each sector depends on the ratio of input rental prices

\[ \frac{k_{N,t}}{l_{N,t}} = \left( \frac{\alpha_N}{\beta_N} \right) \frac{w_t}{u_{M,t}} \]

\[ \frac{k_{T,t}}{l_{T,t}} = \left( \frac{\psi}{1 - \psi} \right) \frac{w_t}{u_{M,t}} \]

As is clear from the above expressions, the degree of capital intensity in each sector is given by \( \left( \frac{\psi}{1 - \psi} \right) \) and \( \left( \frac{\alpha_N}{\beta_N} \right) \). We will show that if the non-traded sector is more labor intensive, that is, \( \left( \frac{\psi}{1 - \psi} \right) > \left( \frac{\alpha_N}{\beta_N} \right) \), there will labor reallocation from the tradable to non-tradable sector in the face of a positive commodity shock and an increase in the overall demand for labor.

### 3.3 Fiscal Rule

In line with the empirical evidence, we assume an active fiscal policy. In particular, the fiscal authority chooses the transfer relative to GDP as a function of profits in the commodity sector,
\[ \hat{v}y_t = \rho v \hat{v}y_{t-1} + \theta_O \hat{\pi}_{O,t} + \theta_y y_{M,t} + \sigma_v \epsilon^v_t \]

where \( v_y = v_t / y_{M,t} \) and \( \epsilon^v_t \) is and standard normal random variable. Hat variables denote deviation relative to its steady-state value. Since the measure we use in the empirical exercise - total income associated to commodity production- is not the same as the model variable, \( \hat{\pi}_{O,t} \), the semi-elasticity parameters \( \rho_v, \theta_O, \theta_M \) and \( \sigma_v \) will be calibrated to match the empirical correlation of \( v_y \) with the price of the commodity and non-commodity GDP observed in the data. As in the empirical specification in Section 2, the cyclical nature of the fiscal rule depends on the signs of \( \theta_O \) and \( \theta_y \).

### 3.4 Competitive Equilibrium

In equilibrium the supply and demand of non-tradable must be equal, so

\[ y_{N,t} = c_{N,t} + i_{O,t} + y_{om} \]

This equation highlights two important mechanisms through which commodity prices affect the non-commodity economy. First, investment in the commodity sector is entirely non-tradable. Second, a constant fraction of the demand for intermediate goods is also non-tradable.

In the tradable sector, the excess demand for goods could be financed if households run trade deficits, \( tb_t \), which are financed with foreign debt, \( d_t \). As a result, the equilibrium in this market could be characterized by the following equations describing the determination of the trade balance, the private debt and the interest rate faced by households:

\[ tb_t = y_{T,t} + v_t + u_{O,t} k_{O,t} + p_{om} y_{om} - c_{T,t} - i_{M,t} - y_{om} \]  
\[ d_t = (1 + r_t - 1) d_{t-1} - tb_t \]
\[ r_t = r^* + \zeta(e^{d_t - \bar{d}} - 1) \]

where, as in [Schmitt-Grohe and Uribe (2003)](Schmitt-Grohe2003), we assume that the interest rate premium paid for foreign debt depends on the deviation of the actual debt level from its steady-state value, \( \bar{d} \).

**Exogenous Shocks.** We assume that the price of the commodity is exogenous and given
by:

\[ \ln(\hat{p}_{O,t}) = \rho_{O}\ln(\hat{p}_{O,t-1}) + \sigma_{O}\epsilon_{O,t} \]  

(3.13)

where \( \rho_{O} \) and \( \sigma_{O} \) are the persistency and standard deviation respectively and the hat denotes the ratio of a variable relative to its steady state. Similarly, the technology shock, which impacts both traded and non-traded sectors, is given by:

\[ \ln(\hat{A}_{j,t}) = \rho_{A}\ln(\hat{A}_{j,t-1}) + \sigma_{A}\epsilon_{A,t}^{A} \]

where \( j = T,N \). Without loss of generality, we assume the same shock process for both sectors.

4 The Quantitative Effects of a Copper Price Shock

In this section we present a quantitative analysis of our model. Our main question is how copper prices propagate to the rest of the economy and the relative importance of copper price volatility in explaining business cycle fluctuations. To this end, we start by describing how the model is calibrated in order to match salient facts of the Chilean economy. Then, we discuss the fit of the model and compare the IRFs of the model with the SVAR. Finally, we perform counterfactual exercises to explore how sensitive the moments are to different features of the model.

4.1 Calibration

**Exogenous coefficients.** The model is set in quarterly terms. As in Schmitt-Grohé and Uribe (2015), we set the risk aversion coefficient, \( \sigma \), to 2. This is a value commonly used in business cycle literature. We set \( \omega = 1.31 \), which implies a Frisch elasticity of labor supply, \( 1/(\omega - 1) \), of 3.2. This value is somehow higher than the one used by Schmitt-Grohé and Uribe (2015) and Mendoza (1991), but enable us to better approximate the volatility of labor observed in the data. We choose an exogenous world risk-free interest rate of 4 percent, which implies a discount factor of \( \beta = 1/(1 + r^*) = 0.96 \). The depreciation rate in both sectors is set to \( \delta_{M} = \delta_{O} = 0.07 \), in line with the relative importance of labor in the non-tradable production function, \( \beta_{N} \), is set to 0.75 in line with values computed for Chile by Medina and Naudon (2012). Incidentally, this value is the same as the one reported for Argentina by Schmitt-Grohé and Uribe (2015), and other previous studies for Chile. We normalize the steady-state level of productivity in the tradable sector to \( A_{T} = 1 \). The value of the steady-state level of productivity in the non tradable sector
is set to $A_N=6.2$ in order to generate a ratio of non-tradable output to non-commodity GDP of 42%\footnote{If we decide to normalize the price of nondurable goods to 1 in steady-state, we could adjust this coefficient and see the relative size of the non-tradable sector.}. The elasticity of substitution between tradable and non-tradable consumption goods, $\chi$, is set to 52%. This value implies that the expenditure of non-tradable consumptions goods, relative to the expenditures in tradable goods, is 0.92. This is close to the value estimated for Chile (0.97).

The relative importance of labor in the non-tradable production function, $\beta_N$, is set to 0.75 in line with values computed for Chile by Medina and Naudon (2012). Incidentally, this value is the same as the one reported for Argentina by Schmitt-Grohé and Uribe (2015). The relative importance of labor in the tradable production function, $1 - \psi$, is set to 0.49 which is slightly larger than the value reported by Medina and Naudon (2012), but similar to the one reported by Uribe (1997) for Argentina. This value is, in any case, such that the tradable sector is more capital intensive than the non-tradable sector. This feature is well documented for Chile and other emerging economies (see Uribe (1997) and Schmitt-Grohé and Uribe (2015)). We set the relative importance of capital in the non-tradable sector, $\alpha_N$, to 0.12. This value implies, given the rest of the coefficients, a steady-state share of non-commodity labor to GDP equal to 0.43 which is in line with the empirical estimates for Chile.

From the 2013 input-output matrix we extract the coefficients that characterize the copper production function. In particular, the share of intermediate goods, $\beta_O$, is 0.2 whereas the relative importance of capital, $\alpha_O$, is set to 0.32.\footnote{From the input-output matrix we can compute the surplus in the mining industry in Chile. This value is 0.5 of total production and includes both rents and payments to capital. In order to compute capital payments we subtract from the surplus foreign investors rents (which we obtained from the balance of payments) and state owned copper company (CODELCO) rents. Based on this, we set $\alpha_O$ to 0.32.} The relative share of tradable and non-tradable goods in the production of intermediate goods, the $a$ and $b$ coefficients, are $a=0.4$ and $b=0.6$.

### Estimated parameters
The price of copper is an exogenous process and shocks to this variable could be interpreted as a structural shock. To determine the size and persistence of this shock, we estimate equation (3.13) using HP-filtered price of copper deflated by world inflation. We obtain values of $\rho = 0.81$ and $\sigma_O = 0.06$.

### Calibrated parameters
Table (3) reports the targets and parameters associated with each moment. We calibrate the level of long-run debt, $\bar{d}$, to match the trade balance to GDP ratio, for the non-commodity sector, to its average value of -7 percent. The coefficient of the...
interest rate premium, $\zeta$, is set to 0.003 in order to match the volatility of the trade balance to GDP in the non-commodity sector. This value is slightly larger than the one set exogenously by Aguiar and Gopinath (2007) and Schmitt-Grohe and Uribe (2003). It is, in any case, a very small value as not to affect the dynamic of the interest rate. The value of the investment adjustment costs, $\phi_M$ and $\phi_O$ are calibrated to match two moments related to investment: the volatility of aggregate investment and the relative volatility of investment in the commodity and non-commodity sectors. Note that, in order to match the empirical moments, investment adjustment costs are substantially larger in the commodity sector, implying that investment in that industry is much more volatile than in the non-commodity sector. The persistence and volatility of the productivity shock, the coefficients $\rho_A$ and $\sigma_A$, are calibrated in order to replicate the autocorrelation and standard deviation of the constant price non-commodity GDP we found in the data. By the same token, the volatility of shocks to the fiscal rule, the $\sigma_{vy}$ coefficient, is set to match the volatility of the data of $vy_t$. Finally, in the case of the fiscal rule, as in Fernández-Villaverde et al. (2015), we assume that this an exogenous process. The persistence of this rule is set as in our empirical estimation, that is, we set $\rho_v=0.31$. The coefficients $\theta_O$ and $\theta_{ym}$ are calibrated to match the correlation of $vy_t$ with commodity GDP and real non-commodity output.

Table 3: Calibration: Benchmark Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{d}$</td>
<td>-5.572</td>
<td>$tb_M/y_M$</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.003</td>
<td>$std(tb_M/y_M)$</td>
<td>1.97</td>
<td>1.97</td>
</tr>
<tr>
<td>$\phi_M$</td>
<td>0.016</td>
<td>$std(i)$</td>
<td>7.09</td>
<td>7.09</td>
</tr>
<tr>
<td>$\phi_O$</td>
<td>0.345</td>
<td>$std(i_o)/std(i_M)$</td>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>0.721</td>
<td>$ac1(y_M)$</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>0.007</td>
<td>$std(y_M)$</td>
<td>2.23</td>
<td>2.23</td>
</tr>
<tr>
<td>$\sigma_{vy}$</td>
<td>0.016</td>
<td>$std(v/y_M)$</td>
<td>2.21</td>
<td>2.21</td>
</tr>
<tr>
<td>$\theta_{cu}$</td>
<td>-0.012</td>
<td>$corr(vy,pOyO)$</td>
<td>-0.38</td>
<td>-0.38</td>
</tr>
<tr>
<td>$\theta_{ym}$</td>
<td>-0.422</td>
<td>$corr(vy,ym)$</td>
<td>-0.59</td>
<td>-0.59</td>
</tr>
</tbody>
</table>

Notes: Parameters are calibrated simultaneously to match the empirical moments generated by the data. Empirical moments are computed with detrended data for Chile from 1996.Q1 to 2013.Q4.
4.1.1 Moments not matched

We now explore moments not matched in our calibration. Table (4) shows a list of steady-state and dynamic moments that we did not calibrate. First, the calibrated model generates reasonable steady-state values. Panel (a) shows that the relative size of the investment in the copper sector relative to non-commodity investment in steady-state is about 0.27, which is very close to the data. Similarly, the investment output ratio in non-commodity is 0.15, which is slightly lower than in the data. Second, the model also has the ability to match some dynamic moments. In panel (b) we show that the model is able to generate a correlation between the price of copper and non-commodity output of 0.31, which is roughly 60 percent of the observed correlation in the data. Moreover, the model produces the right sign and similar magnitude for the negative correlation of the trade balance-non-commodity GDP ratio and the copper price, the positive correlation of aggregate investment and non-commodity GDP and the price of non-tradables and the price of copper.

We also compare the ability of the model to generate the variance decomposition result we reported in section 2. Recall that in the SVAR analysis 44 percent of the variance of non-commodity output is explained by the price of copper. Our model predicts that 19 percent of the variance of non-commodity output is explained by copper price fluctuations. We also compare the model to the fiscal regressions we presented in Section 2. We simulate the variables \( \nu_t, y_M \) and \( y_O \) and estimate the fiscal rule regression using equation (1). In panel (c) we see that these estimates are not statistically different from the ones we obtain with actual data. In particular, the autocorrelation and the coefficient of non-commodity GDP is similar to the estimation. The commodity revenue coefficient falls in the 95 percent confidence interval of the estimation.

4.2 Model Fit

**Impulse responses.** Figure 2 depicts the impulse response function of aggregate variables to a one standard deviation shock to the price of copper. Higher copper prices increase the return of investing in the commodity sector and hence, increases the demand for commodity-specific capital and intermediates. Since the demand for non-tradable commodity investment and intermediates rises, there is a shift in the relative supply of non-tradable goods increasing

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13We perform this exercise because the theoretical rule uses profits from the commodity sector, which is the relevant variable, rather than commodity income, which is the variable we have to use in the empirical exercise.
Table 4: Non-Targeted Moments

<table>
<thead>
<tr>
<th>(a) Steady state</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_M$/Y</td>
<td>0.20</td>
<td>0.27</td>
</tr>
<tr>
<td>$y_M$/$y</td>
<td>0.22</td>
<td>0.15</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>2.30</td>
<td>2.42</td>
</tr>
</tbody>
</table>

(b) Dynamic Moments

- $corr(p_o, y_M)$: 0.55 (0.31)
- $corr(b/y_M, p_o)$: -0.35 (-0.68)
- $corr(i, y_M)$: 0.79 (0.49)
- $corr(p_o, p_N)$: 0.52 (0.60)
- $corr(p_o, l)$: 0.24 (0.17)
- $var_{decomp}(y_M|p_o)$: 0.44 (0.19)

(c) Fiscal regressions

$v_{yt} = \rho v_{yt-1} + \theta_o \tilde{r}_{o,t-1} + \theta_y \tilde{y}_{M,t-1} + \epsilon_t$

<table>
<thead>
<tr>
<th>Estimation (99% conf. interval)</th>
<th>Implied by Model (calibrated)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>[0.005, 0.656]</td>
</tr>
<tr>
<td>$\theta_o$</td>
<td>[-0.0668, -0.0118]</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>[-0.898, 0.150]</td>
</tr>
</tbody>
</table>

NOTE: (a) Steady state ratios constructed as the average of the observed data from 1996 to 2013. In the case of (b) dynamic moments, empirical moments are computed with detrended data for Chile from 1996.Q1 to 2013.Q4. For (c) fiscal regressions, we use the simulated data from our model and estimate a fiscal policy rule like (1). We present the 95% confidence interval in square brackets and compare our results to the calibrated fiscal rule coefficients.

the relative price of non-tradable goods. Interestingly, aggregate labor increases. An increase in $p_N$ induces an increase in the relative demand for labor, generating an increase in both real wages and employment. This, despite the fact that the labor supply shifts to the left (i.e. declines) in the face of higher prices of non-traded goods. Investment in the non-commodity sector falls upon impact. This is because the tradable sector is contracting and, since it is capital intensive, the decline in tradable investment dominates the increase in non-tradable investment. As a result, non-commodity capital stock responds sluggishly. Since aggregate labor increases and capital moves gradually, real output in the non-commodity sector expands after the shock. Finally, the non-commodity trade balance (excluding copper exports) deteriorates for about 10 quarters. Consistent with Schmitt-Grohé and Uribe (2015), the overall trade balance improves upon impact due mainly to the increase in copper exports.
NOTE.- This figure plots the responses of commodity investment ($inv_O$), the price of non-traded goods ($pN$), non-commodity employment ($labor$), non-commodity investment ($inv_M$), GDP in the non-commodity sector ($y_M$) and non-commodity trade balance to GDP ($tby_M$).

**Reallocation between sectors.** A terms of trade shock has often been compared with an aggregate productivity shock. It turns out that both shocks generate similar aggregate responses but different reallocation implications. In Figure 3 we compare the responses of several variables to an aggregate productivity shock (shock to both sectors) and to a copper price shock. The solid line display the response to a copper price shock. As before, non-commodity GDP and the relative price of non-tradable goods increases. This rise in prices increases the value of the marginal productivity of both labor and capital in the traded sector. As a consequence, resources (inputs) move to the non-tradable sector for any given level of capital and labor in the economy. Now, besides the reallocation, there are changes in the supply of labor and capital. In particular, since the non-tradable sector is labor intensive, the expansion of this sector increases the total demand for labor. Because the labor supply is elastic, total employment and wages
increase. By the same token, the contraction of the tradable sector, which is capital intensive, generates a decline in non-commodity capital and investment. As a result, there is a decline in the relative size of the tradable sector. This is a Dutch disease phenomenon explained mainly by investment/intermediate demand instead of the traditional spending effect (as in Pieschacón (2012)). Given the fall in relative prices, there is a relative shift in consumption from non-tradable towards tradable goods. As a consequence, production of tradable declines and tradable consumption increases, leading to a deterioration of the trade balance.

The reallocation between sectors resembles the classical Rybczynski theorem from trade theory. An increase in aggregate demand leads to an increase in the non-tradable output because it uses labor intensively. This is indeed reinforced by an increase in the price of the non-traded good, which makes non-tradable production more profitable.

4.3 Comparing the IRF with the SVAR

In Figure (4) we compare the impulse response of the model with those from the SVAR estimation. We observe that the model is able to replicate reasonably well the response of key aggregate variables to a copper price shock. In particular, the model generates an expansion in the non-commodity output and aggregate investment, a real exchange rate appreciation and a fall in non-commodity trade balance to GDP.

The model also tracks reasonably well the dynamic response of the variables. In particular, the dynamic response of non-commodity GDP and labor, investment, the real exchange rate and the non-commodity trade balance are in line with those of the SVAR. Most of the response is explained by a sluggish reaction of investment demand in the commodity sector that propagates to the rest of the economy.

14Recall that the Rybczynski theorem states that at constant relative goods prices, a rise in the endowment of one factor will lead to a more than proportional expansion of the output in the sector which uses that factor intensively, and an absolute decline of the output of the other good.
Figure 3: Comparing the IRF of Price of Copper and aggregate TFP

NOTE.- This Figure depicts IRFs to a one standard deviation shock in the price of commodity ($pcu$) and a common technology shock ($\epsilon^A_t$). The size of each shock is given by the calibrated values in Table 3.

Figure 4: Comparing IR from the Model and the SVAR.

NOTE.- This Figure depicts IRFs to a one standard deviation shock in the price of commodity in both the SVAR (data in blue) and the benchmark model (in red). Responses are percentage deviations from steady-state values.
4.4 Importance of the Investment Channel and the Fiscal Rule

We have shown that the model performs relatively well to match the data. To gain more intuition about which feature of the model is explaining different moments we perform two counterfactual experiments. Table (5) shows a comparison of the benchmark estimation with two alternative counterfactual exercises.

Effect of non-tradable investment and intermediates. First, in column (I) we shut down the investment and intermediate channels by setting $\alpha_O$ and $\beta_O$ to zero. We observe that this feature is important for the model to generate a correlation of the price of copper with non-commodity output and real exchange rate. Without non-tradable investment and intermediate demand, the model generates a counterfactual negative correlation between investment and the price of copper. Finally, without this mechanism the contribution of the price of copper to non-commodity output variance is nearly zero.

Effect of the fiscal rule. In column (II) we remove the fiscal rule equation from the model. We see that without the rule the model generates higher non-commodity output volatility. The rule dampens the effect of the price of copper on activity.

Table 5: Counterfactual Scenarios

<table>
<thead>
<tr>
<th>Moments</th>
<th>Benchmark</th>
<th>(I) $\alpha_0 = 0, \theta_O = 0, \theta_M = 0$</th>
<th>(II) $\beta_0 = 0, \rho_v = 0, \sigma_v = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$std(y_M)$</td>
<td>2.23</td>
<td>2.08</td>
<td>3.11</td>
</tr>
<tr>
<td>$std(tb/y_M)$</td>
<td>1.97</td>
<td>1.33</td>
<td>1.53</td>
</tr>
<tr>
<td>$ac(tby_M)$</td>
<td>0.53</td>
<td>0.04</td>
<td>0.85</td>
</tr>
<tr>
<td>$corr(p_o, y_M)$</td>
<td>0.31</td>
<td>-0.06</td>
<td>0.37</td>
</tr>
<tr>
<td>$corr(p_o, p_N)$</td>
<td>0.60</td>
<td>-0.09</td>
<td>0.65</td>
</tr>
<tr>
<td>$corr(i, p_o)$</td>
<td>0.54</td>
<td>-0.15</td>
<td>0.68</td>
</tr>
<tr>
<td>$corr(l, p_o)$</td>
<td>0.17</td>
<td>-0.06</td>
<td>0.27</td>
</tr>
<tr>
<td>$corr(v/y_M, y_M)$</td>
<td>-0.59</td>
<td>-0.53</td>
<td>nd</td>
</tr>
<tr>
<td>$corr(v/y_M, p_o y_o)$</td>
<td>-0.38</td>
<td>-0.13</td>
<td>nd</td>
</tr>
<tr>
<td>$vardecomp(y_M</td>
<td>p_o)$</td>
<td>0.19</td>
<td>0.03</td>
</tr>
</tbody>
</table>

NOTE.-: The benchmark moments are the ones calibrated and non calibrated in our baseline scenario (Tables 3 and 4). In scenario I, we assume commodity output is an endowment which does not require intermediates capital nor intermediates and hence, $\alpha_O = \beta_O = 0$. In the case of scenario II we assume there is no fiscal rule, so $\theta_O = \theta_M = \rho_v = \sigma_v = 0$. In both scenarios, the rest of the coefficients are the same as in the benchmark calibration.
5 Alternative Models

In this section we consider alternative models and compare their ability to explain the data with our benchmark. We require all models to match the same moments. In this way we give each alternative model a fair chance to fit the same set of moments. We then compare moments not matched and impulse responses across all models. We consider two alternative specifications. In the first scenario we remove the non-tradable sector from the model. In the second one, we remove fiscal policy. In particular, we set the rule coefficients to zero, that is, $\theta_O = \theta_M = \rho_v = \sigma_v = 0$. In both scenarios we recalibrate the relevant coefficients, so comparison with our benchmark model reflects the relevance of each channel.

Table 6: Calibration: Benchmark Parameters

<table>
<thead>
<tr>
<th>Moments</th>
<th>Benchmark</th>
<th>(I) Only Tradable</th>
<th>(II) No Fiscal Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibrated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$tb_M/y_M$</td>
<td>-0.07</td>
<td>-0.07</td>
<td>-0.07</td>
</tr>
<tr>
<td>std($tb_M/y_M$)</td>
<td>1.97</td>
<td>1.97</td>
<td>1.97</td>
</tr>
<tr>
<td>std(i)</td>
<td>7.09</td>
<td>7.09</td>
<td>7.09</td>
</tr>
<tr>
<td>std($i_o)/std(i_M$)</td>
<td>3.70</td>
<td>3.70</td>
<td>3.70</td>
</tr>
<tr>
<td>$ac_1(y_M)$</td>
<td>0.83</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>std($y_M$)</td>
<td>2.23</td>
<td>2.23</td>
<td>2.23</td>
</tr>
<tr>
<td>std(v/y_M)</td>
<td>2.21</td>
<td>2.21</td>
<td>na</td>
</tr>
<tr>
<td>corr(v/y_M, y_M)</td>
<td>-0.59</td>
<td>-0.59</td>
<td>na</td>
</tr>
<tr>
<td>corr(v/y_M, p_Oy_O)</td>
<td>-0.38</td>
<td>-0.38</td>
<td>na</td>
</tr>
<tr>
<td>Non-calibrated</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ac_1(tby_M$)</td>
<td>0.53</td>
<td>0.19</td>
<td>0.70</td>
</tr>
<tr>
<td>corr(p_o, y_M)</td>
<td>0.31</td>
<td>-0.20</td>
<td>0.37</td>
</tr>
<tr>
<td>corr(p_o, p_N)</td>
<td>0.60</td>
<td>na</td>
<td>0.68</td>
</tr>
<tr>
<td>corr(i, p_o)</td>
<td>0.45</td>
<td>-0.32</td>
<td>0.45</td>
</tr>
<tr>
<td>corr(l, p_o)</td>
<td>0.17</td>
<td>-0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>vardecom($y_M$</td>
<td>p_O)</td>
<td>0.19</td>
<td>0.27</td>
</tr>
</tbody>
</table>

NOTE.- The benchmark moments are the ones calibrated and non calibrated in our base scenario (Tables 3 and 4). In scenario I, we assume non-commodity output is tradable with a given exogenous price. In this case the commodity sector uses inputs that can be produced domestically or imported. In the case of scenario II we assume there is no fiscal rule, so $\theta_O = \theta_M = \rho_v = \sigma_v = 0$. In both scenarios, coefficients are recalibrated in order to match the same moments that in the benchmark case.
5.1 The Role of the Non-Tradable Sector

In this scenario we assume that non-commodity GDP is tradable. As a consequence, the price of domestically produced goods is given in the international markets. In this scenario, given that the countercyclical policy rule is in place, a positive shock to the price of commodities generates a reduction in $v_y$. This, in turn, induces a decline in consumption which translates into a decline in non-commodity GDP. Because the price of tradable goods is given, there is no price adjustment that could attenuate the decline in non-commodity GDP. Hence, in this scenario the demand for both labor and capital (investment) in the non-commodity sector shrinks. The demand for intermediates and capital in the commodity sector increases, although with non substantial impact on domestic GDP. This is so, because both investment and intermediate goods could be imported (they are tradable). As a consequence, as shown in Figure 5, we observe a decline in non-commodity GDP, a reduction in labor and investment, and also a deterioration of the trade balance. As is clear, all these features are at odds with the data despite the fact that we have calibrated the model to match the same target moments as in our benchmark specification (see Table 6). In this scenario the only channel through which the commodity price shock is transmitted to the economy is the countercyclical fiscal rule.\footnote{If we keep the assumption that there is a tradable and a non-tradable sector, but we model the commodity as an endowment ($\alpha_O = \beta_O = 0$), we obtain similar results: non-commodity GDP and employment decline, whereas the investment’s reaction is close to zero.}
Figure 5: Mainland is only Tradable.

5.2 No Fiscal Rule

To assess the impact of the fiscal rule in place, we modify our benchmark scenario and set the rule coefficients to zero, that is, $\theta_O = \theta_M = \rho_v = \sigma = 0$. In this case a positive commodity price shock generates an expansion of the commodity sector, increasing the demand for commodity-specific non-tradable capital and intermediate goods. As a consequence, the demand for non-traded goods and the their price ($p_N$) increase. Given the fact that this sector is labor intensive, the demand for labor expands along with the expansion in non-tradable production. Aggregate investment increases, given the expansion in the commodity-specific investment and the relatively
mild contraction in non-commodity investment and capital. Now, removing the fiscal rule tends to exacerbate the positive correlation between the price of commodity and non-commodity GDP, investment, labor, and the price of non-tradable goods (see Table 6 and Figure 6). As expected, in this scenario it is not possible to match the negative correlation between $\nu y_t$ and both the price of commodity and non-commodity GDP.

![Figure 6: No Fiscal Rule.](image)

**NOTE.** Responses of GDP non-commodity, non-commodity labour, investment, price of non-traded goods and non-commodity trade balance to a commodity price shock. Blue dotted line: responses in the data (SVAR), red line: responses in the benchmark calibration (Table 3 parametrization) and green dotted line: responses under a model in which there is no fiscal rule.

16 There is a reallocation of capital across sectors. In particular, capital moves from the traded to the non-traded sector. Overall non-commodity capital and investment decline, but not as much as traded capital.
6 Conclusions

In this paper we have documented that commodity price shocks have a significant impact on the business cycle of a small commodity producer, Chile, in spite of a countercyclical fiscal policy. In particular, using SVAR analysis for Chile we find that an increase in the price of copper leads to an increase in both non-commodity GDP and labor, a real appreciation and a decrease in non-commodity trade balance over GDP. Moreover, our variance decomposition analysis shows that the price of copper constitutes 40 percent of the variance of non-copper GDP.

To determine what are the mechanisms that could explain the stylized facts, we derive a quantitative three-sector real business model where a commodity is produced using non-tradable investment and intermediate goods. We calibrate the model using several steady-state and dynamic moments to reflect realistic features of the Chilean economy. We find that our model has a good fit to several unconditional moments and dynamic responses to a rise in the price of copper. In our model an increase in the price of commodities is transmitted to the rest of the economy through two different, but complementary, channels. First, the reallocation channel is such that positive commodity price shocks increase the value of the marginal productivity of both labor and capital in the non-traded sector. As a consequence, resources (inputs) move to the non-tradable sector for any given level of capital and labor in the economy (i.e. this reallocation channel will be present even if labor and capital are in fixed supply). Second, there is an endogenous response in factor supply that enables the non-tradable sector to expand beyond what will be its level if only the reallocation channel is present. In particular, given the fact that the non-tradable sector is labor intensive, the expansion of this sector increases the total demand for labor. Because the labor supply is elastic, total employment and wages increase. By the same token, the contraction of the tradable sector, which is capital intensive, generates a decline in non-commodity capital and investment. As a result of these two mechanisms, there is a decline in the relative size of the tradable sector. Finally, we show that omitting any of these factors leads to counterfactual dynamic responses and to unconditional correlations that differs from the ones implicit in the data.
References


7 Appendix

7.1 Households:

Households’ first-order optimality conditions with respect to $c_{T,t}, c_{N,t}, l_t, d_t, i_{M,t+1}, i_{O,t+1}, k_{M,t+1}, k_{O,t+1}$:

\[c_{T,t} : \quad (c_t - l_t^\omega / \omega)^{-\sigma} c_t^{1-\chi} = \lambda_t\] (7.1)

\[c_{N,t} : \quad (c_t - l_t^\omega / \omega)^{-\sigma} (1 - \chi) c_t^{1-\chi} = \lambda_t p_{N,t}\] (7.2)

\[l_t : \quad (c_t - l_t^\omega / \omega)^{-\sigma} l_t^{\omega-1} = \lambda_t w_t\] (7.3)

\[d_t : \quad \beta(1 + r_t) E_t \lambda_{t+1} = \lambda_t\] (7.4)

\[i_{M,t+1} : \quad \lambda_t = -\mu_{M,t}(\phi M(\frac{i_{M,t}}{i_{M,t-1}} - 1) \frac{i_{M,t}}{i_{M,t+1}} - [1 - \frac{\phi M}{2}(\frac{i_{M,t}}{i_{M,t-1}} - 1)^2])\] (7.5)

\[i_{O,t+1} : \quad \lambda_{O,t} = -\mu_{O,t}(\phi O(\frac{i_{O,t}}{i_{O,t-1}} - 1) \frac{i_{O,t}}{i_{O,t+1}} - [1 - \frac{\phi O}{2}(\frac{i_{O,t}}{i_{O,t-1}} - 1)^2])\] (7.6)

\[k_{M,t+1} : \quad \mu_{M,t} = \beta(u_{M,t} + \mu_{M,t+1}(1 - \delta_m))\] (7.7)

\[k_{O,t+1} : \quad \mu_{O,t} = \beta(u_{O,t} + \mu_{O,t+1}(1 - \delta_O))\] (7.8)

Relative demand for consumption: From the household first-order conditions, divide (7.1) by (7.2) to obtain:

\[\frac{c_{N,t}}{c_{T,t}} = \left(\frac{1 - \chi}{\chi}\right) \frac{1}{p_N}\] (7.9)

Labor Supply:

7.2 Firms

\[k_{T,t} : \quad A_{T,t}\psi(l_{T,t}/k_{T,t})^{1-\psi} = u_{M,t}\] (7.10)

\[l_{T,t} : \quad A_{T,t}(1-\psi)(k_{T,t}/l_{T,t})^\psi = w_t\] (7.11)

\[k_{N,t} : \quad p_{N,t} A_{N,t} \alpha_N(k_{N,t}^{\alpha_N-1})(\frac{\alpha_N}{N_{N,t}})^{\alpha_N} = u_{M,t}\] (7.12)

\[l_{N,t} : \quad p_{N,t} A_{N,t} \beta_N(k_{N,t}^{\beta_N-1})(\frac{\beta_N}{N_{N,t}})^{\beta_N} = w_t\] (7.13)
7.3 Driving forces (shocks)

\[
\begin{align*}
\log(A_{T,t}) &= (1 - \rho_{T}) \log(A_{T}) + \rho_{T} \log(A_{T,t-1}) + \epsilon_{T,t} \\
\log(A_{N,t}) &= (1 - \rho_{N}) \log(A_{N}) + \rho_{N} \log(A_{N,t-1}) + \epsilon_{N,t} \\
\log(A_{O,t}) &= (1 - \rho_{O}) \log(A_{O}) + \rho_{O} \log(A_{O,t-1}) + \epsilon_{O,t} \\
\log(p_{O,t}) &= (1 - \rho_{p}) \log(A_{pO}) + \rho_{P} \log(p_{O,t-1}) + \epsilon_{pO,t}
\end{align*}
\]

7.4 Closing the economy

To close the economy, we impose the following constraints:

\[
\begin{align*}
l_{t} &= l_{T,t} + l_{N,t} \\
\pi_{t}^{O} &= p_{O,t}y_{O,t} - u_{O,t}k_{O,t} - p_{l}^{om}y_{l}^{om} - q_{t}S \\
p_{l}^{om} &= a + bp_{N,t} \\
y_{l}^{om} &= ay_{l}^{om} \\
y_{N,t}^{om} &= by_{N,t}^{om} \\
y_{N,t} &= c_{N,t} + i_{O,t} + y_{N,t}^{om} \\
t_{b} &= y_{T,t} + v_{t} + u_{O,t}k_{O,t} + p_{l}^{om}y_{l}^{om} + q_{t}S - c_{T,t} - i_{M,t} - y_{T,t}^{om} \\
d_{t} &= (1 + r_{t-1})d_{t-1} - t_{b} \\
k_{M,t} &= k_{T,t} + k_{N,t} \\
p_{M,t} &= (y_{T,t} + p_{N,t}y_{N,t})/(y_{T,t} + y_{N,t})
\end{align*}
\]

where equations (7.20) to (7.29) impose equilibrium in the labor market, defines profits of oil industry, defines the price of intermediate input used by oil sector, close the non-tradable sector, close the tradable sector (defining trade balance), defines the evolution of debt, defines non-commodity capital and non-commodity prices, respectively.

Also, we have to impose some conditions that give consistency to the model. These are:
\[ y_{M,t} = \left( y_{T,t} + p_N y_{N,t} \right)/p_{M,t} \]  \hspace{1cm} (7.30)
\[ y_t = y_{M,t} + y_{O,t} \] \hspace{1cm} (7.31)
\[ i_t = i_{M,t} + i_{O,t} \] \hspace{1cm} (7.32)
\[ \frac{tb_t}{y_t} = \frac{tb_t}{y_t} \] \hspace{1cm} (7.33)
\[ k_t = k_{M,t} + k_{O,t} \] \hspace{1cm} (7.34)
\[ k_{M,t+2} = k_{M,t+1} \] \hspace{1cm} (7.35)
\[ k_{O,t+2} = k_{O,t+1} \] \hspace{1cm} (7.36)

where (7.30)-(7.36) define non-commodity GDP, total GDP, total investment, trade balance over GDP, total capital and the auxiliary variables for capital.

### 7.5 Steady state

The deterministic steady state is characterized by the following equations:

\[ r = r^* \] \hspace{1cm} (7.37)
\[ d = \bar{d} \] \hspace{1cm} (7.38)
\[ A_T = \overline{A_T} \] \hspace{1cm} (7.39)
\[ A_N = \overline{A_N} \] \hspace{1cm} (7.40)
\[ A_O = \overline{A_T} \] \hspace{1cm} (7.41)
\[ p_O = \overline{p_O} \] \hspace{1cm} (7.42)
\[ v = \theta_c \] \hspace{1cm} (7.43)

The rest of the variables in steady-state could be computed as follows:

1. Guess a value of non tradable price \( p_N \).

2. Using (7.7) and (7.8) (imposing the steady-state condition) find:

\[ \mu_M = \lambda \] \hspace{1cm} (7.44)
\[ \mu_O = \lambda p_N \] \hspace{1cm} (7.45)

3. In S.S from equations (7.7) and (7.8) we get:

32
\[ u_M = \frac{\mu_M}{\lambda} \left( 1 - \frac{\beta(1 - \delta_M)}{\beta} \right) \]  
\[ (7.46) \]

\[ u_O = \frac{\mu_O}{\lambda} \left( 1 - \frac{\beta(1 - \delta_O)}{\beta} \right) \]  
\[ (7.47) \]

1. Dividing (7.10) by (7.11) to obtain the ratio:

\[ \frac{k_T}{l_T} = \left( \frac{\psi}{1 - \psi} \right) \frac{w}{u_M} \]  
\[ (7.48) \]

Replace this last expression in (7.11) to obtain wage:

\[ w = \left[ (1 - \psi) A_T \left( \frac{\psi}{1 - \psi} \frac{1}{u_M} \right) \right]^{\frac{1}{1-\psi}} \]  
\[ (7.49) \]

2. From the household first-order conditions, divide (7.1) by (7.2) to obtain:

\[ \frac{c_N}{c_T} = \left( \frac{1 - \chi}{\lambda} \right) \frac{1}{p_N} \]  
\[ (7.50) \]

Then replace \( \lambda \) from (7.2) in (7.3), and using the previous result obtain aggregated steady-state labor as:

\[ l = \left[ w\chi(c_N/c_T)^{1-\chi}\right]^{1/(\omega-1)} \]  
\[ (7.51) \]

3. From the non tradable first order conditions, divide (7.13) by (7.12) to obtain:

\[ k_N = \left( \frac{\alpha_N}{\beta_N} \right) \left( \frac{w}{u_M} \right) l_N \]  

Replacing this last expression into (7.12) and re-arranging terms:

\[ l_N = (p_N A_N)^{\frac{1}{1-\sigma_N}} \left( \frac{\alpha_N}{u_M} \right)^{\frac{\alpha_N}{1-\sigma_N-\sigma_N}} \left( \frac{\beta_N}{w} \right)^{\frac{1-\alpha_N}{1-\sigma_N-\sigma_N}} \]  
\[ (7.52) \]
Replacing (7.52) in (7.12) we obtain:

\[ k_N = (p_N A_N)^{-\frac{1}{\alpha_N-\beta_N}} \left( \frac{\alpha_N}{u_M} \right)^{\frac{1-\beta_N}{1-\alpha_N-\beta_N}} \left( \frac{\beta_N}{w} \right)^{\frac{\beta_N}{1-\alpha_N-\beta_N}} \]  (7.53)

4. Using (7.20), (7.51) and (7.52) we obtain tradable labor:

\[ l_T = l - l_N \]  (7.54)

Using (7.48) we obtain tradable capital:

\[ k_T = \left( \frac{\psi}{1 - \psi} \right) \frac{w}{u_M} l_T \]  (7.55)

5. Solve the system of equations (7.14) and (7.15) and obtain:

\[ y_{om} = p_O A_O S^{\gamma_O} \left( \beta_O \right)^{1-\alpha_O} \left( \frac{\alpha_O}{w_O} \right)^{\frac{\alpha_O}{1-\alpha_O-\beta_O}} \]  (7.56)

\[ k_O = \left( \frac{p_{om}(y_{om})^{1-\beta_O}}{p_O A_O \beta_O S^{\gamma_O}} \right)^{\frac{1}{\beta_O}} \]  (7.57)

6. Using (3.8)-(3.7) obtain GDP:

\[ y_T = A_T k_T^{\psi} l_T^{1-\psi} \]  (7.58)

\[ y_N = A_N k_N^{\alpha_N} l_N^{\beta_N} \]  (7.59)

\[ y_O = A_O k_O^{\alpha_O} (y_{om})^{\beta_O} \]  (7.60)

From the properties of the Leontief technology obtain:

\[ y_T^{om} = a y_{om} \]  (7.61)

\[ y_N^{om} = b y_{om} \]  (7.62)
7. Obtain non-commodity capital, investment, oil profits, trade balance and tradable/non tradable consumption as follows:

\[ k_M = k_T + k_N \]  \hspace{1cm} (7.63)
\[ i_O = \delta_O k_O \]  \hspace{1cm} (7.64)
\[ i_M = \delta_M k_M \]  \hspace{1cm} (7.65)
\[ \pi^O = p_O y_O - u_O k_O - \hat{p}^o m y^o m - qS \]  \hspace{1cm} (7.66)
\[ t_b = r d \]  \hspace{1cm} (7.67)
\[ c_N = y_N - i_O - y^o m \]  \hspace{1cm} (7.68)
\[ c_T = y_T + \pi^O + v - t_b - i_M - y^o m \]  \hspace{1cm} (7.69)

8. We compute the measurement error of the model as the difference between (7.50) and the ratio of (7.68) to (7.69). If the difference is small, you have the non-tradable price. Else, find a new guess and repeat all previous steps. Once the algorithm converges, compute the rest of auxiliary variables of interest.