Trend Inflation and Wicksellian Rules: A Remedy for Indeterminacy *

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Abstract

We show that a contemporaneous Wicksellian monetary policy rule, in which the policy rate reacts to deviations of the current price level and current output gap, has a determinacy region that is invariant to the level of trend inflation. This is because the price level converges to a deterministic path under such rule, which implies that the expectations of future inflation are well anchored irrespectively of the level of trend inflation. A Taylor rule, in contrast, has a determinacy region that shrinks as the level of trend inflation increases.

Keywords: Wicksellian rules, Taylor rules, determinacy.

JEL codes: E31, E52, E58, C62.

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I Introduction

Since the seminal work of Taylor (1993), Taylor rules have received widespread attention in the empirical and theoretical literature on monetary policy. The reason behind this is that they are generally considered realistic yet sufficiently simple representations of how the monetary authority conducts policy.\(^1\) In a nutshell, Taylor rules relate in a linear fashion the monetary policy rate to deviations of the inflation rate and a measure of the output gap.

As it is well known in the literature, Taylor rules are able to induce determinacy.\(^2\) Depending on whether the rule reacts to lagged, contemporaneous, or expected inflation and output, determinacy conditions may change across alternative Taylor rules. In general, however, determinacy requires a common constraint for all rules, which is the so-called Taylor principle. This principle states that the policy rate should react by more than the increase in inflation in the long run, so as to ensure that the real interest rate increases. To the extent that this principle prevents the emergence of multiple equilibria, it is considered a desirable feature of any given monetary policy rule.

Ascari and Ropele (2009) show that even moderate levels of trend inflation modify the conditions under which the rational expectations equilibrium (REE) is determinate or unique. In particular, the long-run Phillips curve is positively sloped around the zero-inflation steady state; however, as soon as trend inflation takes up even moderate positive values, the long-run Phillips curve inverts and becomes negatively sloped, reflecting the pernicious effect of relative price distortions on output. In other words, the higher the level of trend inflation, the lower the level of output in steady state. This implies that when trend inflation increases, implementable monetary rules call for increasingly larger and positive coefficients on inflation and smaller coefficients on output. Consequently, as trend inflation increases, the determinacy region shrinks, increasing the possibility of sunspot fluctuations.

In this paper we show that contemporaneous Wicksellian rules, in which the policy rate reacts to deviations of the current price level and current output gap, have a determinacy region that is invariant to the level of trend inflation and determinacy conditions are satisfied as long the policy response to prices is positive and the policy reaction to the output gap is equal or greater than zero.

\(^1\)See, among others, Clarida et al. (2000) for evidence in the USA, Clarida et al. (1998) and Lubik and Schorfheide (2007) for estimation of Taylor rules in advanced economies and Aizenman et al. (2011) for evidence in emerging countries.

II Determinacy regions and trend inflation

We closely follow Ascari and Ropele (2009) and consider a version of the New Keynesian model with positive trend inflation. In this setup, we introduce two alternative policy rules, a contemporaneous Taylor rule and a Wicksellian rule in the spirit of Giannoni (2014).

The contemporaneous Taylor rule is expressed as:

\[ i_t = \phi_\pi \pi_t + \phi_x x_t, \]

where \( i_t \) is the monetary policy rate, \( \pi_t \) is the inflation level and \( x_t \) is the output gap. The policy response coefficients are \( \phi_\pi \) and \( \phi_x \). In this case the matrix representation of the standard New Keynesian model considered in Ascari and Ropele (2009) with indivisible labor and no indexation to past inflation is given by:

\[ y_t = B E_t y_{t+1} + C z_t, \]

where \( y_t = [\pi_t, \phi_t, x_t] \), \( z_t = [r_t^e; u_t] \) and

\[
B = \begin{pmatrix} 1 & 0 & -\lambda \\ 0 & 1 & 0 \\ \phi_\pi & 0 & 1 + \phi_x \end{pmatrix}^{-1} \begin{pmatrix} \beta \Pi^{1-\epsilon} + \eta(\theta - 1) & \eta & 0 \\ (\theta - 1)\alpha \beta \Pi^{(\theta-1)(1-\epsilon)} & \alpha \beta \Pi^{(\theta-1)(1-\epsilon)} & 0 \\ 1 & 0 & 1 \end{pmatrix},
\]

with

\[
\lambda = \frac{[1 - \alpha \Pi^{(\theta-1)(1-\epsilon)}][1 - \alpha \beta \Pi^{(\theta-1)}}{\alpha \Pi^{(\theta-1)(1-\epsilon)}}
\]

and

\[
\eta = \beta(\Pi^{1-\epsilon} - 1)[1 - \alpha \Pi^{(\theta-1)(1-\epsilon)}],
\]

where \( \alpha \) is the Calvo probability of not optimizing prices in the current period, \( \beta \) is the discount factor, \( \theta \) is the elasticity of substitution, \( \epsilon \) is the degree of indexation to trend inflation and \( \Pi \) is one plus the level of trend inflation. The variable \( \phi_t \) is an auxiliary variable. When the approximation is around a steady state characterized by positive inflation, the IS and Taylor rule equations remain unchanged.\(^3\) The main difference arises in the case of the New Keynesian Phillips curve. In this case, the structural coefficients depend on the level of trend inflation.

\(^3\)See Ascari and Sbordone (2014).
The determinacy conditions under a Taylor rule also depend on $\bar{\Pi}$. As trend inflation increases, a given value for the policy response to output, $\phi_x$, requires a more aggressive response to inflation, $\phi_\pi$, to ensure determinacy. This is reflected in one of the determinacy conditions under the Taylor rule:

$$\phi_\pi + \phi_x \delta(\bar{\pi}, \epsilon) > 1,$$

where $\delta(\bar{\pi}, \epsilon)$ is the long-run elasticity of output to inflation. As a consequence, in the plane $(\phi_x, \phi_\pi)$ the determinacy region shrinks as trend inflation increases. This is because $\delta(\bar{\pi}, \epsilon)$ switches sign from positive to negative as soon as trend inflation increases over a given threshold.

A contemporaneous Wicksellian rule can be expressed as:

$$i_t = \psi_p p_t + \psi_x x_t,$$

where $\psi_p$ and $\psi_x$ represent the magnitude of the reaction of the nominal interest rate to deviations of the output gap and the price level.\(^5\,^6\)

Under this Wicksellian rule the system under trend inflation contains an additional state variable, $p_{t-1}$, so $y_t = [\pi_t, \phi_t, x_t, p_{t-1}]$. In this case, the matrix representation of the standard New Keynesian model considered in Ascari and Ropele (2009) is given by:

$$B = \begin{pmatrix}
1 & 0 & -\lambda & 0 \\
0 & 1 & 0 & 0 \\
\psi_p & 0 & 1 + \psi_x & \psi_p \\
1 & 0 & 0 & 1
\end{pmatrix}^{-1} \begin{pmatrix}
\beta \bar{\Pi}^{1-\epsilon} + \eta(\theta - 1) & \eta & 0 & 0 \\
(\theta - 1)\alpha\beta \bar{\Pi}^{(\theta-1)(1-\epsilon)} & \alpha\beta \bar{\Pi}^{(\theta-1)(1-\epsilon)} & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}.$$

We conjecture that the determinacy conditions under a Wicksellian rule like (3) depend only on the sign of the policy response coefficients. In particular, determinacy is ensured as long as the following conditions are satisfied:

$$\psi_p > 0$$

\(^4\)See Ascari and Ropele (2009) for details on this and the other relevant determinacy conditions.\(^5\)Notice that, as we are working with variables expressed in deviations from the steady state, the variable $p_t$ can be interpreted as the deviation of the price level from a predetermined target that, in the case of positive trend inflation, grows at the trend rate.\(^6\)As shown by Giannoni (2014), under contemporaneous Wicksellian rules such as (3) and no trend inflation, all possible values of $\psi_p > 0$ and $\psi_x \geq 0$ yield a rational expectations equilibrium that is unique (a determinate system). Bauducco and Caputo (2018) show that this result extends to backward-looking, forward-looking and hybrid Wicksellian rules.

\(^{3}\)}
and

\[ \psi_x \geq 0. \tag{5} \]

In order to prove our conjecture, we compute numerically the areas of determinacy under the two alternative policy rules in the presence of trend inflation. We follow the parameterization of Ascari and Ropele (2009) and set \( \alpha = 0.75, \beta = 0.99, \theta = 11, \epsilon = 0.5 \). We consider three values of trend inflation, \( \bar{\Pi} = \{0, 1\%, 3\%\} \). Figures 1, 2 and 3 show the regions of determinacy of Taylor and Wicksellian rules for \( \bar{\Pi} = 0, \bar{\Pi} = 1\% \) and \( \bar{\Pi} = 3\% \), respectively. Notice that the case in which \( \bar{\Pi} = 0 \) is our benchmark specification (no trend inflation). We only include it here to show that the model with trend inflation encompasses the simpler model with no trend inflation analyzed in the main text.

It is clear from Figures 1-3 that the determinacy areas generated by the contemporaneous Wicksellian rule are invariant to the level of trend inflation, i.e., the only necessary condition to achieve determinacy is that the coefficients \( \psi_p > 0 \) and \( \psi_x \geq 0 \). The Taylor rule, on the other hand, presents areas of determinacy that shrink clockwise with higher levels of trend inflation, as explained by Ascari and Ropele (2009). The intuition is simple: as trend inflation causes the long-run Phillips curve slope to become (more) negative, the nominal interest rate has to increase by more in response to permanent inflation to ensure stability. This, of course, is the Taylor principle at work. In the case of the Wicksellian rule, which entails the converge of the price level to a deterministic path, the expectations of future inflation are well anchored irrespectively of the level of trend inflation. Therefore, an increase in the nominal interest rate in the presence of inflation is sufficient to ensure that the real interest rate increases as well.

Hence, we prove, by means of this numerical example, that contemporaneous Wicksellian rules are immune, in terms of their determinacy properties, to introducing trend inflation in the model.

References


Figure 1: Determinacy areas of contemporaneous rules - $\bar{\Pi} = 0$

Note: The darker area corresponds to combinations of parameters \( \{\phi_x, \phi_\pi\} \) of the Taylor rule and parameters \( \{\psi_x, \psi_p\} \) of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters \( \{\phi_x, \phi_\pi\} \) of the Taylor rule for which the system is indeterminate, but to parameters \( \{\psi_x, \psi_p\} \) of the Wicksellian rule for which the system is determinate.

Figure 2: Determinacy areas of contemporaneous rules - $\bar{\Pi} = 1\%$

Note: The darker area corresponds to combinations of parameters \( \{\phi_x, \phi_\pi\} \) of the Taylor rule and parameters \( \{\psi_x, \psi_p\} \) of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters \( \{\phi_x, \phi_\pi\} \) of the Taylor rule for which the system is indeterminate, but to parameters \( \{\psi_x, \psi_p\} \) of the Wicksellian rule for which the system is determinate.

Figure 3: Determinacy areas of contemporaneous rules - $\bar{\Pi} = 3\%$

Note: The darker area corresponds to combinations of parameters \( \{\phi_x, \phi_\pi\} \) of the Taylor rule and parameters \( \{\psi_x, \psi_p\} \) of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters \( \{\phi_x, \phi_\pi\} \) of the Taylor rule for which the system is indeterminate, but to parameters \( \{\psi_x, \psi_p\} \) of the Wicksellian rule for which the system is determinate.