Wicksellian rules and the Taylor principle: some practical implications

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Abstract

In this paper we derive and compare the determinacy regions of price-level targeting rules (called Wicksellian rules) and Taylor rules in a standard new-Keynesian model. We conclude that Wicksellian rules, under different specifications, do not require the Taylor principle to be satisfied in order to induce determinacy. Our results have two implications. First, we show that, in a univariate setting, estimating simple Taylor rules when the true rule is Wicksellian may lead to erroneously conclude that the equilibrium is indeterminate even if the true data generating process is such that determinacy is ensured. Second, if the policy estimation is performed using system based methods, indeterminacy is ruled out. However, the policy misspecification will lead to conclude that the central bank is less averse to inflation movements than it actually is.

Keywords: Wicksellian rules, Taylor rules, determinacy, estimation.

JEL codes: E31, E52, E58, C62.

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1 Introduction

Since the seminal work of Taylor (1993), Taylor rules have received widespread attention in the empirical and theoretical literature on monetary policy. The reason behind this is that they are generally considered realistic yet sufficiently simple representations of how the monetary authority conducts policy.\(^1\) In a nutshell, Taylor rules relate in a linear fashion the policy rate to deviations of the inflation rate and a measure of the output gap. In recent years price-level targeting (PLT) rules, in which the policy rate reacts to deviations of the *price level* instead of inflation, have regained attention among academics and policymakers.\(^2\),\(^3\) As in Giannoni (2014), we refer to PLT rules as Wicksellian rules.\(^4\)

The objective of this paper is twofold. First, in a standard New Keynesian model, we derive the areas of determinacy of forward-looking, contemporaneous and hybrid Wicksellian rules (rules that react to the expected level of prices and contemporaneous output gap). Second, we assess the implications of estimating Taylor rules in a context in which the central bank is setting its policy based on Wicksellian rules. The purpose of this exercise is to determine the extent to which one can reach misleading conclusions about the nature of monetary policy if one fails to recognize that the monetary authority is setting policy according to a Wicksellian rule.

As it is well known in the literature, Taylor rules are able to induce determinacy.\(^5\) Depending on whether the rule reacts to lagged, contemporaneous, or expected inflation and output, determinacy conditions may change across alternative Taylor rules. In general, how-

\(^1\)See, among others, Clarida et al. (2000) for evidence in the USA, Clarida et al. (1998) and Lubik and Schorfheide (2007) for estimation of Taylor rules in advanced economies and Aizenman et al. (2011) for evidence in emerging countries.

\(^2\)Several elements have contributed to this fact. Gorodnichenko and Shapiro (2007) show that the Fed, in the Volcker-Greenspan era, was behaving consistently with having the price level in its objectives. They also concluded that price-level targeting (PLT) is superior to inflation targeting (IT) in a wide range of situations. Further evidence is provided by Bullard (2012) who noticed that prices in the USA have fluctuated, between 1995 to 2012, around a 2% price path. Expanding the sample from 1991 to 2014 reinforces this finding (see Figure 8 in the Appendix). Reis (2013) reaches a similar conclusion. In the case of Canada, Kamenik et al. (2013) and Ruge-Murcia (2014) provide similar evidence to the one reported in Figure 8. In particular, they show that, since the mid 1990s, in Canada consumer prices, inflation expectations and the policy rate are determined in a way that is consistent with an element of price-level-targeting.

\(^3\)In a recent contribution, Giannoni (2014) concludes, in the context of a New Keynesian model, that simple Wicksellian rules perform better in terms of welfare and are more robust to alternative shock processes than Taylor rules. Evans (2012), on the other hand, argues that PLT would be a helpful complement of Fed’s strategies in the aftermath of the 2008 global financial crisis.

\(^4\)For a deeper discussion on the specific proposal by Knut Wicksell and the price stabilization policies in the 1930s in Sweden see Jonung (1979) and Berg and Jonung (1999).

ever, determinacy requires a common constraint for all rules, which is the so-called *Taylor principle*. This principle states that the policy rate should react by more than the increase in inflation, so as to ensure that the real interest rate increases.

Our main findings are as follows. First, in striking contrast with Taylor rules, Wicksellian rules do not require the Taylor principle\(^6\) to be satisfied in order to generate determinacy.\(^7\) Hence, under Wicksellian rules, the policy rate could react less than inflation. This is true regardless of whether the rule reacts to lagged, contemporaneous, or expected inflation and output. The reason for this has to do with the expectations of future changes in prices that Wicksellian rules entail. In particular, suppose that a cost-push shock creates inflationary pressures that drive the price level above its long-run equilibrium. Under a Wicksellian rule, agents know that inflation today will be counteracted by a deflation in future periods, in order for the price level to return to its target level. If the Taylor principle is not satisfied, the initial increase in the policy rate is of a lower magnitude than the increase in the inflation rate. However, expectations of a future deflation are responsible for driving the real interest rate up, and thus stabilizing the economy. In this case, it is expectations of future changes in prices, and not the reaction of the policy rate, what drives the real interest rate in the right direction.

Second, we show that the estimated response to inflation in a simple non-inertial Taylor rule that only reacts to inflation is a downward-biased estimator of the true response to prices if the monetary authority sets its policy based on a Wicksellian rule. The bias is such that the estimated inflation response is half the size of the response to prices in the Wicksellian rule. As a consequence, the econometrician may conclude, erroneously, that the system is not determined if she fails to recognize that the central bank follows a Wicksellian rule. Misleading conclusions regarding the nature of the policy implemented by the central bank emerge also in the case of rules that incorporate a reaction to the output gap, as well as rules that are system-based estimated. These results ring a warning bell for empirical economists that use estimations of Taylor rules to extract conclusions about the nature of monetary policy in different time periods.\(^8\)

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\(^6\)As in Ascari and Ropele (2009) we interpret the Taylor principle as a policy reaction coefficient to inflation which is greater than one and the generalized Taylor principle as the requirement that the nominal interest rate is raised by more than the increase in inflation in the long run.

\(^7\)As Bullard and Mitra (2007) show, super-inertial Taylor rules (that is, Taylor rules of the form \(i_t = \phi_i i_{t-1} + i^*_t\) where \(i^*_t\) is a function of inflation and the output gap, and \(\phi_i \geq 1\)) do not need to satisfy the Taylor principle either. Super-inertial Taylor rules, however, are different from Wicksellian rules in several dimensions, so our implications for Wicksellian rules can not be applied to super-inertial Taylor rules.

\(^8\)See, for example, Clarida et al. (2000).
Despite the renewed interest in price level targeting\textsuperscript{9}, there are only few studies investigating the properties of Wickeslian rules in the context of New Keynesian models, in terms of their ability to stabilize the economy. Some notable examples are Kerr and King (1996) and Woodford (2003), who conclude, in different setups, that if the monetary authority adjusts the interest rate in response to deviations of the price level from a target path, then there is a unique equilibrium under a wide range of parameter choices: all that is required is that the authority raises the nominal rate when the price level is above the target path and lowers it when the price level is below the target path. A similar result emerges in the case in which central banks, besides reacting to prices, also respond to contemporaneous output (see Giannoni (2014)).

The existing studies concentrate almost exclusively on simple rules that react to contemporaneous values of the price level and the output gap. As noted by Bullard and Mitra (2002) contemporaneous data rules place unrealistic informational demands on the monetary authority: precise information on the current level of prices and the output gap are usually not available to policymakers. In the case of Taylor rules, the properties of alternative specifications that consider more realistic informational sets for the central bank have already been derived (Bullard and Mitra (2002) in closed economies and Llosa and Tuesta (2008) small open economies). To the best of our knowledge, our paper is the first to extend this exercise to Wickeslian rules.

The rest of the paper is organized as follows. In Section 2 we derive the determinacy regions of alternative Wickeslian rules and compare them with the areas of analogous Taylor rules. We do so in a standard New Keynesian model. Section 3 studies the implications, in terms of the dynamic path followed by prices and inflation, of implementing either a Wickeslian rule or a Taylor rule. This section also provides evidence suggesting that prices in the U.S. are trend stationary, so it is not possible to reject the hypothesis that the Fed follows a rule that targets the price level. In Section 4 we show that estimating Taylor rules may lead to misleading conclusions regarding the nature of monetary policy, when the central bank actually follows a Wickeslian rule. This results is derived analytically in the case of simple non-inertial rules, and through stochastic simulations in the case of rules that, besides reacting to inflation or prices, introduce policy inertia and a reaction to the output gap. Section 5 concludes.

\textsuperscript{9}See, for instance, Vestin (2006), Gaspar et al. (2007), Billi (2008), Ambler (2009) and more recently Hatcher and Minford (2014).
2 Determinacy regions in a New Keynesian model

The scope of this section is to determine the parameter regions for which a (linearized) standard monetary policy model has a unique solution, under a particular specification of the monetary policy rule followed by the central bank. To this end, we study determinacy properties of different specifications of monetary policy rules in the standard workhorse New Keynesian model analyzed by, among many, Clarida et al. (1999), Woodford (2003) and more recently Giannoni (2014). Since we link our theoretical results to the empirical literature estimating monetary policy rules, we limit our analysis to local determinacy.

The model economy can be represented by two equations, one intertemporal IS equation that is obtained from the intertemporal optimality condition of households, and a forward-looking New Keynesian Phillips curve that summarizes the optimal pricing decision of monopolistically competitive producers that cannot adjust prices every period.

The IS equation is

\[ x_t = E_t x_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_e^e), \]  

where \( x_t = (y_t - y^e_t) \) is the output gap defined as the difference between the level of output, \( y_t \), and its efficient level, \( y^e_t \), \( \pi_t \) is inflation in period \( t \), \( i_t \) is the nominal interest rate, \( r_e^e \) is the efficient interest rate, or the real interest rate of the efficient economy, and \( \sigma \) is the inverse of the intertemporal elasticity of substitution. Each variable is represented as a percentage deviation from its steady state value. For simplicity, trend inflation is set to zero.

We show in Appendix A by means of some examples that our results still hold when the model incorporates a positive level of trend inflation, at least in the case of contemporaneous

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10 We focus our attention in areas where a unique solution exists because areas where there are infinite solutions are areas in which some undesirable outcomes may arise, such as sunspots and equilibria in which fluctuations in inflation and the output gap are driven by self-fulfilling expectations.

11 For an analysis on global determinacy of New Keynesian models with different monetary policy rules, see Benhabib et al. (2001), Benassy (2009) and Adao et al. (2011) among others. Some recent contributions have studied how price-level targeting policies can aid in achieving uniqueness of equilibria: see Mendes (2011) for a discussion on how equilibria only exist for shocks with volatility levels below some threshold with purely forward looking policies (such as non-inertial Taylor rules), while policies with history dependence, such as Wicksellian rules, can restore existence. Ambler and Lam (2015) reinforce this result by showing that price-level targeting eliminates multiplicity of steady-state equilibria and widens the region of determinacy.

12 For simplicity and to deviate as little as possible from existing mainstream literature, we focus our attention to studying determinacy in the neighborhood of a zero-inflation steady state. Models with Taylor rules may display other steady states (see previous discussion of global determinacy) that, despite the fact that they have largely been overlooked in the literature, they potentially can explain phenomena such as Japan’s liquidity trap and lost decade (see Bullard (2010)). We thank a referee for pointing this important issue out.
The New-Keynesian Phillips curve (NKPC) reads

\[ \pi_t = \kappa x_t + \beta E_t\pi_{t+1} + u_t, \]  

(2)

where \( \kappa \) is related to the frequency of price-adjustment by producers, \( \beta \) is the discount factor and \( u_t \) is a cost push shock that introduces a trade-off in the policy maker’s decision problem.\(^{14}\)

To close the model, we need to specify a monetary policy rule to provide a specification for the nominal interest rate. In what follows, we consider simple non-inertial rules, that is, monetary policy rules that depend on variables observed by the central bank and that, consequently, should be feasible to adopt. We compare two different types of rules: rules in which the nominal interest rate reacts to deviations of the output gap and the inflation rate from specified target values, and rules in which the nominal interest rate reacts to deviations of the output gap and the *price level* from target values. The first type of rules are called *Taylor rules*, and they have been widely analyzed in both the theoretical and empirical literature of monetary policy. We follow Giannoni (2014) and refer to the second type of rules as *Wicksellian rules*. Wicksellian rules have received much less attention in the literature of simple rules.\(^{15}\)

In the next subsections, we analytically characterize the regions of determinacy of different specifications for Taylor and Wicksellian rules. In particular, we follow Bullard and Mitra (2002) and consider interest rate equations that react to forward expectations and contemporaneous data. We incorporate in the analysis an additional rule, which we call a *hybrid* rule, that reacts to expected inflation and contemporaneous data. This rule has not received too much attention in the theoretical literature despite the fact that it has been used extensively in empirical research\(^{16}\). In the Appendix we study the empirically less relevant case of rules with lagged data. We follow the calibration by Woodford (1999), Bullard and Mitra (2002) and Giannoni (2014) and set \( \beta = 0.99, \sigma = 0.157 \) and \( \kappa = 0.024. \)

\(^{13}\)Although it is beyond the scope of this paper to show that this is also true for forward, backward and hybrid rules, we conjecture that this is the case, given our results of Appendix A.

\(^{14}\)The shock \( u_t \) represents, for example, changes in distortionary taxes or exogenous variations in the degree of market power of firms. More generally, this shock may be interpreted as the difference between the efficient level of output, \( y_t^e \), and its natural level, \( y_t^N \). The inclusion of such shocks does not alter the results on determinacy of different monetary policy rules.

\(^{15}\)Some exceptions are Giannoni (2014) and Dittmar and Gavin (2005).

2.1 Contemporaneous rules

Contemporaneous rules have been widely used both for theoretical and empirical purposes. From the seminal contribution of Taylor (1993), papers such as Adolfson (2007), Lubik and Schorfheide (2007) and Aizenman et al. (2011) have estimated this type of rules, and many papers and some books, of which Gali (2008) and Gali and Monacelli (2005) are notable examples, have assessed the behavior of such rules in theoretical frameworks.

A contemporaneous non-inertial Taylor rule is represented by

\[ i_t = \phi_\pi \pi_t + \phi_x x_t, \tag{3} \]

where \( \phi_\pi \) and \( \phi_x \) stand for the reaction of the nominal interest rate to deviations of the output gap and the inflation rate from target values.\(^{17,18}\)

As shown by Bullard and Mitra (2002) the necessary and sufficient condition for a rational expectations equilibrium to be unique (that is, for the system to be determinate) is that

\[ \phi_\pi + \frac{1 - \beta}{\kappa} \phi_x > 1. \tag{4} \]

Notice that, from the NKPC (2), a one percent permanent change in inflation implies that the output gap increases by \((1 - \beta)/\kappa\), under the assumption that \( u_t = 0 \). Then, condition (4) states that the increase in the nominal interest rate given a permanent one percent increase in inflation should be higher than one percent. In other words, the nominal interest rate should react by more than the increase in permanent inflation. This guarantees that the real interest rate increases as well, therefore dampening aggregate demand through the IS equation (1) and, consequently, reducing the output gap and inflation.

The contemporaneous Wicksellian rule is

\[ i_t = \psi_p p_t + \psi_x x_t, \tag{5} \]

where \( \psi_p \) and \( \psi_x \) represent the magnitude of the reaction of the nominal interest rate to deviations of the output gap and the price level.\(^{19,20}\)

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\(^{17}\)These coefficients can be determined so as to minimize the loss function of the central bank. In this case, we say that the simple rule is optimal, in the sense that it is the best possible rule given the restriction imposed by the specific functional form of the rule we are assuming. This “optimal simple rule”, however, will in general not implement the first best allocation.

\(^{18}\)For simplicity, we are assuming that the target values for inflation and the output gap are zero.

\(^{19}\)Notice that, as we are working with variables expressed in deviations from the steady state, the variable \( p_t \) can be interpreted as the deviation of the price level from a predetermined target.

\(^{20}\)We are implicitly assuming that the steady state of this economy is an undistorted one, i.e., inflation
Figure 1: Determinacy areas of contemporaneous rules

Note: The darker area corresponds to combinations of parameters \( \{ \phi_x, \phi_\pi \} \) of the Taylor rule and parameters \( \{ \psi_x, \psi_p \} \) of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters \( \{ \phi_x, \phi_\pi \} \) of the Taylor rule for which the system is indeterminate, but to parameters \( \{ \psi_x, \psi_p \} \) of the Wicksellian rule for which the system is determinate.

As shown by Giannoni (2014), under contemporaneous Wicksellian rules such as (5), all possible values of \( \psi_p > 0 \) and \( \psi_x > 0 \) yield a rational expectations equilibrium that is unique (a determinate system). As a result, under contemporaneous Wicksellian rules the policy rate does not need to increase by more than one-for-one with inflation to ensure determinacy.

Notice that a comparison between the areas of determinacy in the space of coefficients \( \{ \phi_x, \phi_\pi \} \) and \( \{ \psi_x, \psi_p \} \) is not immediate, because \( \phi_\pi \) and \( \psi_p \) are coefficients accompanying different variables. They are, however, comparable in the following sense: consider shocks that cause inflation and the output gap to react on impact in the same magnitude under both types of rules. Then, \( \phi_\pi = \psi_p \) implies the same reaction of the nominal interest rate on impact to such shocks. Consequently, a larger determinacy area for combinations of the coefficients of the Wicksellian rule translates into a larger set of actions that the monetary authority can take following a shock that causes a given initial reaction of inflation and the output gap, compared to the case in which the monetary authority follows a Taylor rule. Figure 1 provides a graphical representation of the areas of determinacy for contemporaneous Taylor and Wicksellian rules.

and the output gap are zero. If we considered a model with a different steady state, we would need to define the Wicksellian rule in terms of the deviation of the price level with respect to some deterministic trend for the price level. See Giannoni (2014) for an example.
2.2 Forward-looking rules

In line with Clarida et al. (2000), Woodford (2000), Bullard and Mitra (2002) and Svensson and Woodford (2004) a purely forward-looking Taylor rule can be expressed as:

\[ i_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1}, \]  

(6)

where \( \phi_\pi \) and \( \phi_x \) represent the reaction of the nominal interest rate to deviations of expected inflation and output from target or long-run values\(^{21}\). Bullard and Mitra (2002) show that, in rules such as (6), the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) are\(^{22}\)

\[ \kappa (\phi_\pi - 1) + (1 + \beta) \phi_x < 2\sigma (1 + \beta), \]  

(8)

and

\[ \phi_\pi + \frac{1 - \beta}{\kappa} \phi_x > 1. \]  

(9)

Notice that this rule requires the Taylor principle, equation (9), to be satisfied.\(^{23}\)

A forward-looking Wicksellian rule can be specified as follows

\[ i_t = \psi_p E_t p_{t+1} + \psi_x E_t x_{t+1}, \]  

(10)

where \( \psi_p \) and \( \psi_x \) represent the policy response to deviations of expectations of the price level and to expectations of the output gap, respectively.

The following proposition characterizes the necessary and sufficient conditions for determinacy when rule (10) is implemented.

**Proposition 1.** Under forward-looking Wicksellian rules such as (10), in which \( \psi_x \geq 0, \)

\(^{21}\)As noted by Bernanke and Woodford (1997), the use of this type of rules overcomes problems associated with the long lag between changes in policy and changes in inflation. In particular, the forecast of future inflation, unlike contemporaneous inflation, can be affected by changes in the policy rate.\(^{22}\)Bullard and Mitra (2002) list as an additional condition

\[ \phi_x < \sigma (1 + \beta^{-1}). \]  

(7)

It can be easily shown that (8) and (9) imply (7), so we do not include it as a necessary and sufficient condition.

\(^{23}\)The equilibrium under rule (6) will be unique if \( \phi_\pi \) and \( \phi_x \) are large enough to guarantee that the real rate eventually rises in the face of an increase in inflation. Satisfying the Taylor principle is a necessary, but not sufficient, condition for determinacy. As shown by Bernanke and Woodford (1997), Bullard and Mitra (2002) and Levine et al. (2007) this rule can induce indeterminacy for aggressive responses to inflation and output if condition (8) is not satisfied.
the necessary and sufficient conditions for a rational expectations equilibrium to be unique (determinate) is that

\[ \psi_p > 0 \]  \hspace{1cm} (11)

\[ \kappa (\psi_p - 2) + 2(1 + \beta) \psi_x < 4\sigma (1 + \beta). \]  \hspace{1cm} (12)

Proof. See Appendix D.2.

A natural corollary of the previous proposition is that the Taylor principle is not a condition for determinacy. In particular, in the limiting case in which \( \psi_x = 0 \), determinacy is achieved by an arbitrarily small and positive \( \psi_p \) coefficient (condition (11)). Figure 2 shows that the region for which forward-looking Wicksellian rules are able to induce determinacy is larger than under forward-looking Taylor rules.\(^{24}\)

\(^{24}\)One can see this analytically too: use conditions (8), (9) and (12) to derive the maximum policy response to inflation and prices for a given reaction to the output gap:

\[ \phi_\pi < \frac{2\sigma(1 + \beta) - (1 + \beta)\psi_x}{\kappa} + 1; \quad \psi_p < 2 \left( \frac{2\sigma(1 + \beta) - (1 + \beta)\psi_x}{\kappa} + 1 \right) \]  \hspace{1cm} (13)

It is clear from (13) that for a common policy reaction to the output gap, \( \phi_x = \psi_x \), the maximum response to prices in the Wicksellian rule is twice as large as the maximum response to inflation in the Taylor rule.
2.3 Hybrid rules

In many empirical exercises, Taylor rules are estimated using a specification in which the policy instrument reacts to expected inflation and to the contemporaneous output gap (and some other external variables in the case of open economies). This type of rules can be expressed as:

\[ i_t = \phi_{\pi} E_t \pi_{t+1} + \phi_x x_t, \]  

(14)

As before, the rationale for this specification is that expected inflation, rather than the contemporaneous level of it, could be influenced by monetary policy. This type of rule is usually referred to in the literature as a forward-looking Taylor rule. In order to distinguish this specification from the pure forward-looking one considered in the previous subsection, we call it a hybrid Taylor rule.

As shown by Caputo and Herrera (2013), under hybrid Taylor rules such as (14), the necessary and sufficient conditions for a rational expectations equilibrium to be unique (for the system to be determinate) are that

\[ \kappa (\phi_{\pi} - 1) - (1 + \beta) \phi_x < 2\sigma (1 + \beta), \]  

(15)

and

\[ \phi_{\pi} + \frac{1 - \beta}{\kappa} \phi_x > 1. \]  

(16)

Notice that, in the case of the hybrid Taylor rule, determinacy also requires the Taylor principle to be satisfied (conditions (9) and (16) are identical).

A hybrid Wicksellian rule can be defined as:

\[ i_t = \psi_p E_t p_{t+1} + \psi_x x_t, \]  

(17)

The following proposition characterizes the necessary and sufficient conditions for determinacy when the rule (17) is implemented.

**Proposition 2.** Under hybrid Wicksellian rules such as (17) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that

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26Comparing (8) and (15), it is easy to see that the previous conditions imply a larger area of determinacy for the hybrid rule than for the forward-looking one, but a smaller one than for the contemporaneous rule. This is to be expected, as this rule uses a combination of contemporaneous data and expectations of future variables.
Figure 3: Determinacy areas of hybrid rules

Note: The darker area corresponds to combinations of parameters \( \{ \phi_x, \phi_p \} \) of the Taylor rule and parameters \( \{ \psi_x, \psi_p \} \) of the Wicksellian rule for which the systems are determinate. The lighter shaded area corresponds to combinations of parameters \( \{ \phi_x, \phi_p \} \) of the Taylor rule for which the system is indeterminate, but to parameters \( \{ \psi_x, \psi_p \} \) of the Wicksellian rule for which the system is determinate.

\[
\psi_p > 0, \tag{18}
\]

and

\[
\kappa(\psi_p - 2) - 2(1 + \beta)\psi_x < 4\sigma(1 + \beta). \tag{19}
\]

Proof. See Appendix D.3.

As in the case of forward-looking Wicksellian rules, under hybrid rules the policy rate does not need to increase one-for-one with inflation (condition (18)).

Figure 3 shows the determinacy areas of hybrid rules. When compared to forward-looking rules, the determinacy areas under hybrid rules are much larger. As a consequence, this type of rules can solve the problems related to the poor stabilization properties of inflation and price forecast-based rules.\(^{27}\)

\(^{27}\)Levine et al. (2007) show that an alternative way of expanding the determinacy areas of forward-looking Taylor rules is to specify rules that depend on a discounted sum of current and future rates of inflation, or rules that are expressed in first differences.
3 Wicksellian rules and Taylor rules: dynamic implications

Taylor and Wicksellian rules not only differ in terms of the determinacy areas they induce, as shown in the previous section, but they also imply very different dynamics for prices. Under Wicksellian rules, an increase in the price level beyond the desired price path has to be offset in subsequent periods. Taylor rules do not impose such a restriction, so temporary shocks have a permanent effect on the price level. As a consequence, under any of the Taylor rule specifications discussed in the previous section, the price level is a non-stationary variable, whereas in the case of Wicksellian rules, the price level is stationary.

To understand the different dynamics that prices may follow under alternative monetary rules, it is useful to analyze the solution under each regime. The New Keynesian model with a Taylor rule is such that endogenous variables \( y_t = [x_t; \pi_t] \) are functions of the exogenous, stationary shocks \( r_t^e \) and \( u_t \), i.e., \( y_t = ar_t^e + bu_t \). Under this solution, inflation and the output gap are stationary, but the price level has a unit root. Hence, an exogenous shock generates a permanent shift in the price level. In the case of a Wicksellian rule, there is an additional endogenous predetermined variable, \( p_{t-1} \), so the system has a solution of the form:

\[
y_t = ar_t^e + bu_t + cp_{t-1},
\]

(20)

where \( y_t = [x_t; \pi_t; p_t] \). Since \( c_p < 1 \), as shown by Giannoni (2014), the price level is stationary.

To illustrate the path followed by all variables, we simulate the dynamic response of the economy, characterized by (1) and (2), in response to a cost push shock, assuming that monetary policy can be described by either a contemporaneous Taylor or a contemporaneous Wicksellian rule.\(^{28}\) In the case of the Taylor rule, we set the policy response to inflation to \( \phi_\pi = 1.1 \), so determinacy conditions (and the Taylor principle) are satisfied. Alternatively, in the case of the Wicksellian rule, we impose a small policy response to the price level, \( \psi_p = 0.1 \), which ensures determinacy but implies, on impact, that the Taylor principle is not satisfied.\(^{29}\)

The response of the economy to a cost push shock is presented in Figure 4. In the case of the simple Taylor rule, as expected, the interest rate increases, on impact, more than inflation. As a consequence, the real ex-ante interest rate increases, pushing down the output

\(^{28}\)For simplicity, we set \( \psi_p=\phi_x=0 \) in each rule. The rest of the coefficients in (1) and (2) are, as before, \( \sigma=0.1571, \kappa=0.0238 \) and \( \beta=0.99 \).

\(^{29}\)In fact, in this case the nominal interest rate moves less than one to one with inflation.
gap. The decline in the output gap, in turn, reduces marginal costs, inducing a decline in inflation over time. The price level, on the other hand, increases and does not return to its initial level. Thus, under a Taylor rule, the price level displays a unit root and transitory shocks shift it permanently.

In the case of the Wicksellian rule, the transmission mechanism is quite different. The cost push shock generates, on impact, an increase in the price level and in inflation of the same magnitude. Given the fact that $\psi_p = 0.1$, it follows that the increase in the nominal rate is well below the increase in inflation (see top row of Figure 4). Hence, on impact, the Taylor principle is not satisfied. In subsequent periods, the policy rate remains above its steady state level as long as the price level does not adjust to its initial value. To induce this adjustment, inflation should decline to negative values. This, in turn, requires a contraction in the output gap, which is only possible if the real rate increases. Under a Wicksellian rule, the price level is stationary and it eventually converges to its initial level. In this case, the increase in the real rate is driven, mainly, by expected future deflation. As a consequence,
determinacy is achieved despite the fact that the Taylor principle is not satisfied on impact.\footnote{After the initial period, the policy rate is slightly above its steady state level, whereas inflation is systematically below its long-run value. Consequently, there is a negative relationship between the interest rate and inflation after the initial period, so the Taylor principle is not satisfied in subsequent periods.}

Now, one can argue that Wicksellian rules in our paper and the price level rules in Bullard and Mitra (2007) and Woodford (2003), which emerge as a particular case in which the inertial coefficient $\psi_i = 1$ that accompanies the lagged value of the nominal interest rate $i_{t-1}$, are similar, because in the end both rules react to the price level. It is possible to show, however, that these two types of rules differ in several dimensions. First, price level targeting rules implicit in Bullard and Mitra (2007) and Woodford (2003) are such that the policy rate reacts not only to the output gap (contemporaneous, lagged or expected future level), but also to all past output gaps. This is not the case in our Wicksellian specifications. Second, the determinacy conditions differ substantially across rules. Third, under Wicksellian rules the price level is always stationary and converges to its initial steady state level. In the case of the price level rules implicit in Bullard and Mitra (2007) and Woodford (2003), the price level does not converge to its initial steady state level. Finally, in the presence of trend inflation the determinacy areas of inertial Taylor rules shrinks (see Ascari and Ropele (2009)). In the case of Wicksellian rules, however, the area of determinacy is invariant to the level of trend inflation. As a result, the differences between Wicksellian and standard Taylor rules we have discussed persist even if inertial Taylor rules are considered.

\section{3.1 Is the price level trend-stationary? Evidence for the U.S.}

Taylor and Wicksellian rules deliver stationary processes for inflation and the output gap. These regimes, however, have very different testable implications for the price level. As noted by Ruge-Murcia (2014), a Wicksellian rule generates two alternative predictions. First, since deviations of the price level from the targeted path must be offset in future periods, the price level should follow a stationary process around a deterministic trend; second, deviations from this trend should follow a stationary process around zero. In contrast, under a Taylor rule, the price level displays a unit root. Moreover, the deviations of the price level from trend should display a unit root as well.

The available evidence on the properties of the price level in inflation targeting countries is mixed. On the one hand, Ruge-Murcia (2014) concludes that in Canada the price level is trend stationary and that deviations from this trend follow a stationary process around zero. Similar results for Canada are reported by Kamenik et al. (2013). Therefore, the evidence suggest that, in Canada, the conduct of monetary policy comes closer to a Wicksellian rule
than to a Taylor rule. On the other hand, Ruge-Murcia (2014) shows that in Australia, New Zealand, Sweden and in the United Kingdom, the price level deviates from the path implied by the inflation target.\footnote{A caveat in Ruge-Murcia (2014) is that the price path implied by the inflation target is constructed assuming that the price target at the beginning of the period is equal to the actual price level. As a consequence, results may change if one takes an agnostic view as to whether prices at the beginning of the inflation targeting period differ from the price-target path.}

Despite the fact that prices in the USA seem to be trend stationary\footnote{See Figure 8 in the Appendix and the discussion in Bullard (2012).}, there are, to our knowledge, no formal tests as to whether prices evolve as predicted by a Wicksellian rule. In this context, we test the two main predictions of the model featuring a price-level-targeting rule: i) whether prices are trend stationary in the USA and, if so, ii) if deviations from this trend follow a stationary process around zero. We use monthly data from 1959.01 to 2014.12 for the personal consumption expenditures (PCE) price index. This is the price measure officials chose for their inflation target since 2012.\footnote{On January 25th 2012 Fed Governor Bernanke announced an inflation target of 2\% based on the PCE price index.} In addition, we perform the stationarity test on the GDP deflator series during the same sample period.\footnote{This series is available on a quarterly basis from 1959.Q1 to 2014.Q4.} This price measure has been used in empirical exercises that attempt to characterize monetary policy in the USA.\footnote{See Clarida et al. (2000) and Gorodnichenko and Shapiro (2007).}

Table 1 reports the KPSS stationarity test for the PCE price index and the GDP deflator. For the period 1959.01 to 1979.08 (pre-Volcker era), the null hypothesis of trend stationarity is rejected for both price indices. In the period after the appointment of Volcker as Fed Chairman (the post-Volcker era), price dynamics changed in a significant manner. In particular, from 1979.09 to 2014.12, the hypothesis of trend stationarity cannot be rejected. In this period the slope of the trend, which could be interpreted as the long-run price/inflation target, is 2.46\% in the case of the PCE price index and 2.40\% in the case of the GDP deflator. Furthermore, the hypothesis that the deviation of the price level from trend follows a stationary process around zero cannot be rejected. In the period after the appointment of Greenspan, results are qualitatively similar, although in this case the long-run price/inflation target implicit in the trend is very close to 2\%, which as discussed previously, is the explicit target of the Fed since 2012.\footnote{Our results are robust to the use of alternative unit root tests such as the ADF and the Phillips-Perron test, as well as to considering periods from 1980 to 2014 or 1990 to 2014.}
4 Wicksell or Taylor? Practical implications and a misleading conclusion

As it is well known, since the early 1990’s, central banks in developed and (some) developing economies have progressively adopted inflation targeting regimes. This fact has led to an ongoing interest in the profession to empirically characterize the monetary policy regimes in such countries.

From our discussion in Section 3, when evaluating the monetary policy regime followed by a central bank, a necessary first step should be to assess the dynamic properties of the price level, so as to discern between inflation targeting and price level targeting regimes. As it is clear from Section 3.1, the available evidence for the U.S. suggests that the Fed has followed the latter, at least since the appointment of Volcker as Fed Chairman. Since the seminal contribution of Taylor (1993), however, the profession has consistently estimated inflation-targeting rules in an effort to characterize the policy followed by the Fed.\textsuperscript{37,38}

There are a number of papers that have analyzed the policy of the Fed in different periods of the U.S. monetary policy history based on Taylor rule estimations. Taylor (1999) concludes that in the 1960s and 1970s the U.S. experienced excessive monetary policy ease that led to high levels of inflation around that period. Similarly, Clarida et al. (2000) find that the point estimates for the inflation and output gap coefficients during the pre-Volcker era do not satisfy the Taylor principle and thus fall within the indeterminacy region of a canonical model they use to characterize the U.S. economy. As a consequence, they conclude that the Volcker-Greenspan rule is stabilizing when compared to rules in the previous period. Lubik and Schorfheide (2004) and Boivin (2006) reach a similar conclusion.\textsuperscript{39}


\textsuperscript{38}A notable exception is Gorodnichenko and Shapiro (2007).

\textsuperscript{39}Lubik and Schorfheide (2004) use bayesian techniques to show how the likelihood-based estimation of
The literature previously mentioned characterized monetary policy as a stabilizing one if the Taylor principle is satisfied. However, as we argued in Section 2, the Taylor principle need not be satisfied for the system to be determinate if the central bank follows a price level targeting rule. Hence, estimating a Taylor rule when the central bank actually follows a Wicksellian rule (and the Taylor principle is not satisfied) may erroneously lead one to conclude that monetary policy is not stabilizing. In this section we tackle this issue from three different angles.

### 4.1 Systematic bias in Taylor rules

Estimating a Taylor rule of the form:

\[ i_t = \phi \pi_t + \epsilon_t \]  \hspace{1cm} (21)

may lead to conclude that the system is not determinate if the central bank sets its policy according to:

\[ i_t = \psi_p \pi_t. \]  \hspace{1cm} (22)

If the policy followed by the central bank is (22), it is easy to show that the disturbance term in (21), \( \epsilon_t \), is the lagged interest rate, \( i_{t-1} \). As a consequence, the OLS estimation of \( \phi \pi \) is biased and inconsistent. In particular, the OLS estimator of \( \phi \pi \) is such that:

\[
E \left( \hat{\phi}_\pi \right) = \frac{1}{2} \psi_p + E \left( \frac{\sum_{t=0}^{T} (k_t + z_t)}{\sum_{t=0}^{T} \pi_t^2} \right),
\]

\[
= \frac{1}{2} \psi_p, \]  \hspace{1cm} (23)

where the second term in (23) tends to zero for large samples (i.e. large values of \( T \)).

The following proposition shows this result.
**Proposition 3.** If the central bank follows the policy rule (22), then the OLS estimator of $\phi_\pi$ in (21) is a downward biased estimator of $\psi_p$. The bias is such that $\hat{\phi}_\pi$ tends, asymptotically, to half the value of $\psi_p$. This result is independent of the number and structure (persistence, variance and covariance) of the stochastic shocks in the economy. It is also independent from the structure of the model, as long as the price level follows a stationary process.

**Proof.** See Appendix D.5.

A natural corollary is that there is a range of values for $\psi_p$ that induces determinacy in the system, but generates OLS estimates of $\phi_\pi$ that do not satisfy the Taylor principle:

**Corollary 1.** For values of $\psi_p$ between (0 and 2] the Wicksellian rule in (22) generates a determinate system. However, estimating a Taylor rule like (21) by OLS will lead to the conclusion that the system is indeterminate.

In order to assess how the bias changes for different sample sizes, we simulate the economy characterized by the dynamic IS equation (1), the New Keynesian Phillips curve (2), and the simple Wicksellian rule (22). The stochastic simulation is performed with two different sample sizes, $T = 100$ and $T = 1000$. In each case, we estimate the Taylor rule in (22) using OLS. As expected, for $T = 1000$, the OLS estimator of $\phi_\pi$ is $\frac{1}{2}\psi_p$ (see Table 2). The estimation is such that the null hypothesis of $\hat{\phi}_\pi = \psi_p$ is rejected. For a smaller sample size, $T = 100$ (equivalent to 25 years of data), $\hat{\phi}_\pi$ is still a downward biased estimator of $\psi_p$ and its value is very close to $\frac{1}{2}\psi_p$. Again, in this case the null hypothesis of $\hat{\phi}_\pi = \psi_p$ is rejected (see Table 3).

<table>
<thead>
<tr>
<th>$\psi_p$</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_\pi$</td>
<td>0.050***</td>
<td>0.250***</td>
<td>0.500***</td>
<td>0.550***</td>
<td>0.750***</td>
<td>1.000***</td>
<td>1.500***</td>
<td>2.501***</td>
<td>5.002***</td>
</tr>
<tr>
<td>(0.006)</td>
<td>(0.025)</td>
<td>(0.046)</td>
<td>(0.050)</td>
<td>(0.066)</td>
<td>(0.084)</td>
<td>(0.119)</td>
<td>(0.184)</td>
<td>(0.333)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
<td>1000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.068</td>
<td>0.089</td>
<td>0.104</td>
<td>0.106</td>
<td>0.114</td>
<td>0.125</td>
<td>0.136</td>
<td>0.154</td>
<td>0.183</td>
</tr>
</tbody>
</table>

| * p < 0.1, ** p < 0.05, *** p < 0.001 |

Alternatives to OLS, for instance a two-stage instrumental variables approach, may overcome the bias in (21). However, even if the bias is eliminated, the problem regarding the

---

40We allow for demand and supply shocks, i.e. disturbances to the IS and New Keynesian Phillips curve. We consider i.i.d shocks to $r_t$ and $u_t$, with mean zero and variances $\sigma_{r,s}=1$ and $\sigma_u=0.05$. As before, the rest of the coefficients are as in Giannoni (2014).
identification of the nature of the central bank persists. In particular, if an alternative procedure successfully removes the bias in (21), values of $\psi_p$ in the interval $(0,1]$ will generate estimates of $\phi_{\pi}$ in the same interval, thus violating the Taylor principle. Not surprisingly, as long as the lagged interest rate is omitted from the simple Taylor rule in (21), it would not be possible for the econometrician to properly characterize the monetary policy regime implemented by the central bank.

As noted by Minford et al. (2002), Taylor rules estimates may suffer from simultaneity and endogeneity problems. In what follows, we will consider alternative procedures that aim to tackle those problems. The consequences of policy misspecification, however, persist. In short, Taylor rule estimates do not characterize the nature of the central bank if monetary policy is implemented via a Wicksellian rule.

### 4.2 GMM Estimation

In many empirical applications, an inertial Taylor rule that reacts to expected inflation and to the contemporaneous output gap is specified. In particular, Clarida et al. (1998) and Clarida et al. (2000), among others, estimate for the U.S. and other developed countries a partial adjustment hybrid Taylor rule of the form:

$$i_t = \rho i_{t-1} + \phi_{\pi} E_t \pi_{t+1} + \phi_x x_t + \varepsilon_t. \quad (24)$$

The above rule can be regarded as a misspecified version of the following hybrid Wicksellian rule:

$$i_t = \psi_p E_t \pi_{t+1} + \psi_x x_t. \quad (25)$$

As a consequence, if the true underlying policy governing monetary policy is the hybrid Wicksellian rule (25), estimating a rule such as (24) may lead to misleading conclusions regarding the stabilizing nature of policy implemented by the central bank. To assess the
extent of this problem, we simulate the economy under the hybrid Wicksellian rule (25), and then estimate the Taylor rule in (24). We consider alternative positive values for the $\psi_p$ coefficient and set $\psi_x=2$.

In this particular case, it can be shown that the error term in (24), $\varepsilon_t$, encompasses an endogenous variable $(x_{t-1})$ that is correlated to expected inflation, the output gap and the lagged interest rate. This may generate biased and inconsistent estimates, independently of the sample size $T$. To correct for the bias generated by the correlation between the error term and the explanatory variables, we estimate (24) using an instrumental variable approach as in Clarida et al. (1998) and Clarida et al. (2000). In particular, we remove the unobserved expected inflation deviation by rewriting the policy rule (24) in terms of realized variables as follows:

$$i_t = \rho i_{t-1} + \phi_\pi \pi_{t+1} + \phi_x x_t + \nu_t,$$  \hspace{1cm} (26)

where the error term, $\nu_t$, is a combination of the forecast errors of inflation and the disturbance $\varepsilon_t$. We define a vector of variables $u_t$ within the central bank’s information set, at the time it chooses the interest rate, that is orthogonal to $\nu_t$. Hence, $E[\nu_t|u_t] = 0$. In order to estimate the parameters of interest, we use the generalized method of moments (GMM), instrumenting expected inflation.

Table 4 shows the results of the estimation. Despite the fact that the policy response to expected inflation is positive and statistically different from zero, it is always below one. The policy response to output, on the other hand, is positive for small values of $\psi_p$, but declines and is even negative as $\psi_p$ increases. The persistence coefficient is, in general, not statistically different from zero, and based on the Hansen $J$ test, we cannot reject, in almost any case, the null hypothesis that our instrument set is appropriate.

With policy inertia, the determinacy condition in (16) is modified. In particular the Taylor principle for rule (24) is:

41As before, we generate sequence of stochastic demand and supply i.i.d shocks with mean zero and variances $\sigma_r=1$ and $\sigma_u=0.05$.

42The results that follow are robust to alternative values of the $\psi_x$ coefficient.

43The set of instruments $u_t$ we use includes contemporaneous values of the output gap and inflation, as well as lagged values of inflation. Our results are robust to instrumenting also the output gap, in the event that this variable is not observed by the econometrician.

44We perform a Lagrange-Multiplier (LM) test for underidentification using the Kleibergen and Paap (2006) statistic. This test allows researchers to determine whether the minimal canonical correlation between the endogenous variables and the instruments is statistically different from zero. In this case, the null hypothesis is that the equation is underidentified.
\[ \phi_x + \frac{1-\beta}{\sigma} \phi_x > 1. \]  

(27)

Notice that, for every value of \( \psi_p \), the generalized Taylor principle for hybrid rules, (27), is not satisfied. Consequently, we conclude that the estimated coefficients fall within the indeterminacy area.

Table 4: Hybrid Taylor Rule Estimates (2-Step GMM-IV)

<table>
<thead>
<tr>
<th>( \psi_p )</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \psi_p = 0.1 )</td>
<td>0.092</td>
<td>0.125*</td>
<td>0.132*</td>
<td>0.133*</td>
<td>0.133</td>
<td>0.131</td>
<td>0.124</td>
<td>0.106</td>
<td>0.051</td>
</tr>
<tr>
<td>( \psi_p = 0.5 )</td>
<td>(0.063)</td>
<td>(0.074)</td>
<td>(0.078)</td>
<td>(0.079)</td>
<td>(0.081)</td>
<td>(0.083)</td>
<td>(0.086)</td>
<td>(0.087)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>( \psi_p = 1 )</td>
<td>0.544***</td>
<td>0.603***</td>
<td>0.596***</td>
<td>0.593***</td>
<td>0.583***</td>
<td>0.572***</td>
<td>0.551***</td>
<td>0.514***</td>
<td>0.376*</td>
</tr>
<tr>
<td>( \psi_p = 1.5 )</td>
<td>(0.057)</td>
<td>(0.061)</td>
<td>(0.061)</td>
<td>(0.061)</td>
<td>(0.059)</td>
<td>(0.056)</td>
<td>(0.051)</td>
<td>(0.060)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>( \psi_p = 2 )</td>
<td>0.370***</td>
<td>0.095***</td>
<td>0.024</td>
<td>-0.010</td>
<td>-0.033**</td>
<td>-0.064**</td>
<td>-0.101**</td>
<td>-0.151***</td>
<td></td>
</tr>
<tr>
<td>( \psi_p = 3 )</td>
<td>(0.077)</td>
<td>(0.029)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.013)</td>
<td>(0.011)</td>
<td>(0.008)</td>
<td>(0.006)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>( \psi_p = 5 )</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td></td>
</tr>
<tr>
<td>( \psi_p = 10 )</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td>998</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

J-test | 0.935 | 0.589 | 0.451 | 0.431 | 0.366 | 0.304 | 0.214 | 0.107 | 0.006 |

Standard errors in parentheses
* \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.001 \)

As noted by Kleibergen and Mavroeidis (2009) and Mavroeidis et al. (2014), GMM estimation applied to aggregate data often suffers from weak identification problems. We perform alternative exercises in which the Taylor rules are estimated using the actual simulated data for expected inflation. Even in this case, the misspecification problem persists despite the fact that we have removed the weak instrument problem. In particular, Taylor rule coefficients do not satisfy the Taylor principle and therefore they imply an indeterminate system\(^{45}\).

### 4.3 System-based estimation

We have shown that estimating a Taylor rule in a univariate setting generates downward biased estimates in the case of simple contemporaneous Taylor rules estimated with OLS. When a hybrid Taylor rule is estimated by GMM, as is standard in the empirical literature, the estimated coefficients do not satisfy the Taylor principle, and thus fail to generate determinacy.

To test whether our results are robust to alternative estimation procedures, we use a system based approach to estimate (24). Furthermore, this approach can tackle potential simultaneity problems present in previous methods. As before, the true underlying policy

\(^{45}\)For brevity, we do not report results here, but are available upon request.
governing monetary policy is the hybrid Wicksellian rule (25), whereas the output gap and inflation are determined according to equations (1)) and (2). We set the coefficients in the Wicksellian rule to $\psi_p=1.1$ and $\psi_x=0.5$ and simulate the model with the same sequence of stochastic demand and supply shocks used in the previous exercises (sample size $T=1000$). The coefficients in the dynamic IS and Phillips curve equations are, as before, those used by Giannoni (2014).

As noted by Lubik and Schorfheide (2007) a system-based approach optimally adjusts the estimation of the policy rule coefficients for the endogeneity of the right-hand-side variables. Moreover, it is possible to exploit cross equation restrictions that link agents’ decision rules to the policy parameters. We use Bayesian techniques to estimate the Taylor rule coefficients in (24).

We perform two exercises. First, we estimate the policy rule coefficients $\rho$, $\phi_\pi$, and $\phi_x$ in (24), given the parameters of the rest of the equations of the system, which includes the volatility of shocks, and the simulated data. This exercise shows the impact of misspecification on the Taylor rule coefficients. In the second exercise, we estimate the same Taylor rule coefficients, as well as the volatilities of the demand and supply shocks. In this case, the impact of misspecification goes beyond the policy rule equation and can, eventually, be transmitted to the stochastic processes in the system. The purpose of this last exercise is to assess the extent to which the impact of misspecification on the policy rule coefficients can be reduced (or exacerbated) once we let other coefficients of the system adjust.

We assign prior distributions to Taylor rule coefficients and to the standard deviation of demand and supply shocks, and we use Bayesian techniques to compute the posterior distribution of relevant coefficients. It should be noticed that this system-based approach requires the system to be determinate. As a consequence, the effective prior distribution is truncated at the boundary of the determinacy region. Hence, by construction, the prior and posterior distributions of the policy rule coefficients cannot lie in the indeterminacy region. In short, the Taylor principle always holds.

We choose the density of the priors following Lubik and Schorfheide (2007). The interest rate smoothing coefficient, $\rho$, follows a beta prior distribution with a mean of 0.5 and a standard deviation of 0.25. The priors for $\phi_\pi$ and $\phi_x$ follow a gamma distribution and are centered at 1.1 and 0.5, respectively. The coefficients $\sigma_{r,e}$ and $\sigma_u$ follow an inverse gamma distribution and are centered at their true values, 1 and 0.05 (Table 5). Under the prior mean, the determinacy condition, equation (27), holds. In particular, in the long-run

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46Bayesian estimation of the structural parameters is common in academic and policy circles, for instance in Smets and Wouters (2003), Ireland (2004), Canova and Gambetti (2009), Rabanal and Rubio-Ramirez (2005), and more recently in Lubik and Schorfheide (2007) and Kamenik et al. (2013).
\[ \phi_x + \frac{1 - \beta}{1 - \rho} \phi_x = 2.305 \] which is well above 1, as required by the Taylor principle.

Table 5: Prior Distribution

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Prior Mean</th>
<th>Density</th>
<th>Std. Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>0.500</td>
<td>Beta</td>
<td>0.250</td>
</tr>
<tr>
<td>( \phi_\pi )</td>
<td>1.100</td>
<td>Gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>( \phi_x )</td>
<td>0.500</td>
<td>Gamma</td>
<td>0.250</td>
</tr>
<tr>
<td>( \sigma_{\tau^e} )</td>
<td>1.000</td>
<td>Inv Gamma</td>
<td>4.000</td>
</tr>
<tr>
<td>( \sigma_u )</td>
<td>0.050</td>
<td>Inv Gamma</td>
<td>4.000</td>
</tr>
</tbody>
</table>

Table 6 contains the Bayesian estimates of the Taylor rule coefficients\(^{47}\). The third and fourth columns show results in the case in which all remaining parameters of the model are known. In addition to 90% posterior probability intervals, we also report the posterior mean. The degree of persistence increases substantially, from the 0.5 value of the prior mean, to 0.722. The estimated response to inflation declines significantly, from 1.1 to 0.223, whereas the response to the output gap increases just marginally from 0.5 to 0.546. For all coefficients the 90% interval is very narrow, suggesting, perhaps erroneously, that the data is quite informative in order to identify the policy rule coefficients\(^{48}\). The estimated Taylor rule implies a weaker response to inflation, both in terms of its own prior mean, as well as relative to the estimated response to the output gap. In terms of determinacy, our results show that, in the long-run, \[ \phi_x + \frac{1 - \beta}{1 - \rho} \phi_x = 1.008, \] so the Taylor principle in (27) holds just marginally.

The last two columns of Table 6 show the estimates of the policy coefficients, as well as the volatility of structural shocks. As before, the misspecification of the policy rule tends to affect the estimation of the Taylor rule coefficients. In particular, the posterior mean of the policy response to inflation falls to 0.04, whereas the estimated response to the output gap increases to 0.718. The degree of policy persistence increases further to nearly 0.9. The policy misspecification has also important consequences for the other equations in the

\(^{47}\) The bayesian procedure works as follows. First, using standard optimization routines, we find the posterior mode, which is the most likely in the posterior distribution. Then we use the Metropolis-Hastings algorithm to generate draws from the posterior. This algorithm is implemented with 5,000 draws and four chains. The scale parameter for the jumping distribution’s covariance matrix of the Metropolis-Hastings algorithm is set, in each case, to a value such that the average acceptance rate per chain is close to 30%.

\(^{48}\) To verify that the estimation algorithms work well, we estimate the Wicksell coefficients \( \psi_p \) and \( \psi_x \). Not surprisingly, the posterior mean is 1.1 for \( \psi_p \) and 0.51 for \( \psi_x \) with a very narrow 90% interval. In this case, the log data density is 885.
### Table 6: Parameter Estimation Results: Taylor Rule Posteriors

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Prior Mean</th>
<th>Taylor Rule</th>
<th>Posterior Mean</th>
<th>90% Interval</th>
<th>Taylor Rule/Shocks</th>
<th>Posterior Mean</th>
<th>90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.500</td>
<td>0.722</td>
<td>[0.720, 0.724]</td>
<td>0.891</td>
<td>[0.886, 0.897]</td>
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<td></td>
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<tr>
<td>$\phi_\pi$</td>
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<td>0.223</td>
<td>[0.223, 0.224]</td>
<td>0.040</td>
<td>[0.039, 0.041]</td>
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<tr>
<td>$\phi_x$</td>
<td>0.500</td>
<td>0.546</td>
<td>[0.545, 0.548]</td>
<td>0.718</td>
<td>[0.714, 0.723]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>1.000</td>
<td>-</td>
<td>-</td>
<td>2.238</td>
<td>[2.162, 2.315]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_u$</td>
<td>0.050</td>
<td>-</td>
<td>-</td>
<td>0.030</td>
<td>[0.029, 0.031]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Log Data Density | -1687 | 160

- As previously discussed, the system-based approach employed in this section forces the coefficients of the Taylor rule to lie within the determinacy region. It should be noticed, however, that the estimation delivers a corner solution, in the sense that the estimated coefficients satisfy the Taylor principle marginally, and therefore lie at the boundary of the determinacy region. The estimated response of the policy rate to inflation is, as before, downward biased.

### 5 Conclusions and future research

In a standard New Keynesian model, we compare the areas of determinacy of two alternative instrument rules. The first one is a simple Taylor rule in which the policy rate reacts to changes in inflation and output. The second one is a Wicksellian rule in which the interest rate reacts to movements in the price level and output.

In the context of DSGE models, the impact that misspecification has on the proper estimation of structural coefficients is discussed in Canova and Gambetti (2009).
Our main findings are as follows. First, Wicksellian rules do not need to satisfy the Taylor principle in order to induce determinacy. In particular, such rules are able to generate determinacy even in cases in which the nominal rate reacts less than one to one with inflation. In those cases, the main mechanism ensuring determinacy, or an increase in the real ex-ante interest rate in the face of supply shocks, is the expected future deflation which a Wicksellian rule generates. This fact implies that the area of determinacy for the policy coefficients is, in general, larger for Wicksellian rules than for Taylor rules. Moreover, we show by means of a numerical example that the determinacy areas of contemporaneous Wicksellian rules are immune to the introduction of trend inflation in the model. This is not the case for Taylor rules: their determinacy areas shrink when trend inflation increases. This finding reinforces our results. We leave for future work its generalization.

Second, estimating a Taylor rule when the true underlying rule is Wicksellian, may lead to conclude, erroneously, that the equilibrium is indeterminate even in the case in which the price level rule ensures determinacy. To show this, we derive the theoretical bias of the OLS estimator of the coefficient of a Taylor rule when monetary policy is conducted by means of a Wicksellian rule. The bias is such that the estimated inflation response is half the size of the response to prices in the Wicksellian rule, so the policy response to inflation is severely downward-biased in the Taylor rule. We confirm this result under alternative estimation procedures such as GMM and system-based bayesian techniques by use of simulated data.

Our results suggest that, at least in principle, researchers should be cautious when inferring the stabilizing or destabilizing nature of monetary policy from estimates of Taylor rules. We leave for future research the generalization of these results to richer environments such as open economies.

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50 See Ascari and Ropele (2009).
A Appendix: Determinacy regions with trend inflation and contemporaneous rules

In this section, we closely follow Ascari and Ropele (2009) and consider a version of the New Keynesian model with positive trend inflation and partial indexation to it. As in the main text, in which there is no trend inflation, we consider two types of monetary policy rules in order to close the model: a typical contemporaneous non-inertial Taylor rule, and a contemporaneous Wicksellian rule.

The linearized model with a Taylor rule can be written as:

\[ y_t = B E_t y_{t+1} + C z_t, \]

where \( y_t = [\pi_t; \phi_t, x_t] \), \( z_t = [r^*_t; u_t] \) and

\[
B = \begin{pmatrix}
1 & 0 & -\lambda \\
0 & 1 & 0 \\
\phi_\pi & 0 & 1 + \phi_x
\end{pmatrix}^{-1}
\begin{pmatrix}
\beta \bar{\Pi}^{1-\epsilon} + \eta (\theta - 1) & \eta & 0 \\
(\theta - 1) \alpha \beta \bar{\Pi}^{(\theta - 1)(1-\epsilon)} & \alpha \beta \bar{\Pi}^{(\theta - 1)(1-\epsilon)} & 0 \\
1 & 0 & 1
\end{pmatrix}
\]

with

\[
\lambda = \frac{[1 - \alpha \bar{\Pi}^{(\theta - 1)(1-\epsilon)}][1 - \alpha \beta \bar{\Pi}^{(1-\epsilon)}]}{\alpha \bar{\Pi}^{(\theta - 1)(1-\epsilon)}}
\]

\[
\eta = \beta (\bar{\Pi}^{1-\epsilon} - 1)[1 - \alpha \bar{\Pi}^{(\theta - 1)(1-\epsilon)}]
\]

and where \( \alpha \) is the Calvo probability of not optimizing prices in the current period, \( \beta \) is the discount factor, \( \theta \) is the elasticity of substitution, \( \epsilon \) is the degree of indexation to trend inflation and \( \bar{\Pi} \) is one plus the level of trend inflation.

When the monetary policy rule is a contemporaneous Wicksellian rule, now \( y_t = [\pi_t; \phi_t, x_t, p_{t-1}] \) and

\[
B = \begin{pmatrix}
1 & 0 & -\lambda & 0 \\
0 & 1 & 0 & 0 \\
\psi_p & 0 & 1 + \psi_x & \psi_p \\
1 & 0 & 0 & 1
\end{pmatrix}^{-1}
\begin{pmatrix}
\beta \bar{\Pi}^{1-\epsilon} + \eta (\theta - 1) & \eta & 0 & 0 \\
(\theta - 1) \alpha \beta \bar{\Pi}^{(\theta - 1)(1-\epsilon)} & \alpha \beta \bar{\Pi}^{(\theta - 1)(1-\epsilon)} & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

We follow the parameterization of Ascari and Ropele (2009) and set \( \alpha = 0.75, \beta = 0.99, \)
\[ \theta = 11, \ \epsilon = 0.5. \] We consider three values of trend inflation, \( \bar{\Pi} = \{0, 1\%, 3\%\} \). Figures 5, 6 and 7 show the regions of determinacy of Taylor and Wicksellian rules for \( \bar{\Pi} = 0 \), \( \bar{\Pi} = 1\% \) and \( \bar{\Pi} = 3\% \), respectively. Notice that the case in which \( \bar{\Pi} = 0 \) is our benchmark specification (no trend inflation). We only include it here to show that the model with trend inflation encompasses the simpler model with no trend inflation analyzed in the main text.

It is clear from Figures 5-7 that the determinacy areas generated by contemporaneous Wicksellian rules are invariant to the level of trend inflation, i.e., the only necessary condition to achieve determinacy is that the coefficients \( \psi_p > 0 \) and \( \psi_x \geq 0 \). Taylor rules, on the other hand, present areas of determinacy that shrink clockwise with higher levels of trend inflation, as explained by Ascari and Ropele (2009). The intuition is simple: as trend inflation causes the long-run Phillips curve slope to become (more) negative, the nominal interest rate has to increase by more in response to permanent inflation to ensure stability. This, of course, is the Taylor principle at work.

By means of this numerical example, we can conclude that contemporaneous Wicksellian rules are immune, in terms of their determinacy properties, to introducing trend inflation in the model.
Figure 6: Determinacy areas of contemporaneous rules - $\bar{\Pi} = 1\%$

Note: The darker area corresponds to combinations of parameters $\{\phi_x, \phi_{\pi}\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters $\{\phi_x, \phi_{\pi}\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.

Figure 7: Determinacy areas of contemporaneous rules - $\bar{\Pi} = 3\%$

Note: The darker area corresponds to combinations of parameters $\{\phi_x, \phi_{\pi}\}$ of the Taylor rule and parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the systems are determinate. The lighter area corresponds to combinations of parameters $\{\phi_x, \phi_{\pi}\}$ of the Taylor rule for which the system is indeterminate, but to parameters $\{\psi_x, \psi_p\}$ of the Wicksellian rule for which the system is determinate.
Figure 8: Personal Consumption Expenditure Price Index and 2% Price Path: 1991-2014

B Appendix: Evolution of PCE in the U.S.

C Appendix: backward-looking rules

Under backward-looking rules, the policy maker adjusts the policy rate in response to deviations of the lagged values of the variables of interest with respect to their target values. Taylor rules that react to lagged inflation and output gap, in general, are not considered in the theoretical literature on monetary policy, with the exception of Bullard and Mitra (2002). These rules, however, have been used in studies that attempt to estimate the actual rules used by central banks, such as Rotemberg and Woodford (1999) and Giannoni and Woodford (2004).

A backward-looking Taylor rule can be expressed as follows:

\[ i_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1}, \] (28)

The following propositions, derived in Appendix D.4, characterize the necessary and
sufficient conditions for the existence of a unique equilibrium\textsuperscript{51}.

**Proposition 4.** Under backward-looking Taylor rules such as (28) the necessary and sufficient conditions for a rational expectations to be unique (determinate) are either that

\[ \kappa(\phi_{\pi} - 1) + \phi_x(1 + \beta) > 2\sigma(\beta + 1), \]  
\[ \phi_{\pi} + \frac{1 - \beta}{\kappa} \phi_x < 1. \]  

or

\[ \kappa(\phi_{\pi} - 1) + \phi_x(1 + \beta) < 2\sigma(\beta + 1), \]  
\[ \phi_{\pi} + \frac{1 - \beta}{\kappa} \phi_x > 1. \]  

As it is clear from the proposition, if conditions (29) and (30) hold, the Taylor principle does not need to be satisfied to induce determinacy. On the contrary, if conditions (31) and (32) hold, the Taylor principle is relevant and the area of determinacy is identical to the determinacy area in forward-looking rules (conditions (31) and (32) are identical to conditions (8) and (9)).

A backward-looking Wicksellian rule reads:

\[ i_t = \psi_p p_{t-1} + \psi_x x_{t-1}, \]  

**Claim 1.** Under backward-looking Wicksellian rules such as (33) the necessary and sufficient condition for a rational expectations equilibrium to be unique (determinate) is that

\[ \psi_p > 0 \]  
\[ \kappa(\psi_p - 2) + 2(1 + \beta)\psi_x < 4\sigma(1 + \beta). \]  

In the case of backward-looking Wicksellian rules, the conditions for determinacy, equations (34) and (35)), are also identical to the conditions for forward-looking rules (equations (11) and (12))).

We conclude that, as in the case of forward-looking rules, the determinacy area is considerably smaller than in the case of contemporaneous and hybrid rules (see Figure 9). In the

\textsuperscript{51}The necessary and sufficient conditions for determinacy are not derived by Bullard and Mitra (2002), who only present the set of sufficient conditions. Figure 1 in Bullard and Mitra (2002), however, shows the necessary and sufficient conditions for determinacy.
case of backward-looking rules, an overly aggressive response to prices (inflation) and output induces explosive solutions. Finally, as in the case of forward-looking and hybrid rules, the maximum response to prices in the Wicksellian rule is twice as large as the maximum response to inflation in the backward-looking Taylor rule.

D Appendix: proofs

D.1 Necessary and sufficient conditions for determinacy

In order to prove Proposition 1 and Proposition 2, we need to determine if a certain matrix \( \tilde{B} = B^{-1} \) has exactly two eigenvalues outside the unitary circle\(^{52} \). To this end, we follow Proposition C.2 in Woodford (2003) that lists a set of necessary and sufficient conditions for this to be the case. For ease of exposition, we reproduce this proposition here.

Let the characteristic polynomial of the 3x3 matrix \( \tilde{B} \), be defined as

\[
P(\lambda) = \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0,
\]

where \( \lambda \) are the eigenvalues of \( \tilde{B} \). This equation has one root inside the unit circle and two roots outside if and only if:

\(^{52}\text{Matrix } \tilde{B} \text{ will be subsequently defined in each case of interest.}\)
• Case I

\[ P(1) = 1 + A_2 + A_1 + A_0 < 0, \]  
(36)

and

\[ P(-1) = 1 + A_2 - A_1 + A_0 > 0; \]  
(37)

or

• Case II

\[ P(1) = 1 + A_2 + A_1 + A_0 > 0, \]  
(38)

\[ P(-1) = 1 + A_2 - A_1 + A_0 < 0, \]  
(39)

and

\[ A_4 = A_0^2 - A_0 A_2 + A_1 - 1 > 0, \]  
(40)

or

• Case III

Conditions (38) and (39) hold in addition to

\[ A_4 = A_0^2 - A_0 A_2 + A_1 - 1 < 0 \]  
(41)

and

\[ |A_2| > 3. \]  
(42)

D.2 Determinacy of forward-looking Wicksellian rules

The system composed by equations (1), (2) and (10) can be written as

\[ y_t = B E_t y_{t+1} + C z_t, \]
where \( y_t = [x_t; \pi_t; p_{t-1}] \), \( z_t = [r_t^e; u_t] \) and

\[
B = \begin{pmatrix}
\frac{\sigma \beta + \kappa (1 - \psi_p)}{(\sigma - \psi_x) \beta} & -\frac{1 - \psi_p (1 + \beta)}{(\sigma - \psi_x) \beta} & \frac{\psi_p}{\sigma - \psi_x} \\
-\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\
0 & 1 & 1
\end{pmatrix}^{-1}
\] (43)

This system has two endogenous non-predetermined variables, \( x_t \) and \( \pi_t \), and one endogenous predetermined variable, \( p_{t-1} \). For the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the matrix \( B \) has two eigenvalues inside the unit circle and one outside it.

To simplify the algebra, we define \( \tilde{B} = B^{-1} \), so to achieve determinacy, \( \tilde{B} \) needs to have two eigenvalues outside the unit circle.

The characteristic polynomial of matrix \( \tilde{B} \) is

\[
P(\lambda) = \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0,
\]

where \( \lambda \) are the eigenvalues of \( \tilde{B} \) and

\[
A_2 = -\frac{(\sigma - \psi_x) (1 + \beta) + \sigma \beta + \kappa (1 - \psi_p)}{(\sigma - \psi_x) \beta},
\]

\[
A_1 = \frac{\sigma (2 + \beta) + \kappa - \psi_x}{(\sigma - \psi_x) \beta},
\]

\[
A_0 = -\frac{\sigma}{(\sigma - \psi_x) \beta}.
\]

As a result, we can compute \( P(1) \) and \( P(-1) \) to obtain

\[
P(1) = \frac{\kappa \psi_p}{(\sigma - \psi_x) \beta},
\] (44)

\[
P(-1) = \frac{2(1 + \beta) (\psi_x - 2\sigma) - \kappa (2 - \psi_p) (\sigma - \psi_x) \beta}{(\sigma - \psi_x) \beta}.
\] (45)

We restrict our attention to values of \( \psi_x \geq 0 \). In this case, the sign of \( P(1) \) and \( P(-1) \) depends on whether \( (\sigma - \psi_x) \) is positive or negative. We consider each case separately.

**D.2.1 Case I: \( \sigma < \psi_x \)**

Notice that, in this case, \( P(1) < 0 \) and \( P(-1) > 0 \) if the following conditions are satisfied:
$$\psi_p > 0$$ \hspace{1cm} (46)

$$\kappa(\psi_p - 2) + 2(1 + \beta)\psi_x < 4\sigma(1 + \beta).$$ \hspace{1cm} (47)

Hence, if \( \sigma < \psi_x \) the system is determinate if condition (46) and (47) are satisfied.

**D.2.2 Case II and III: \( \sigma > \psi_x \)**

Notice that, in this case, \( \mathcal{P}(1) > 0 \) and \( \mathcal{P}(-1) < 0 \) hold if conditions (46) and (47) are satisfied. To have determinacy, however, it is required in addition that \( \mathcal{A}_4 > 0 \) (Case II) or \( \mathcal{A}_4 < 0 \) and \( |\mathcal{A}_2| > 3 \) (Case III).

Now, notice that:

$$\mathcal{A}_4 = \frac{\psi_x \beta(\sigma - \psi_x)(\beta - 1) + (\sigma - \psi_x)\beta\kappa + \psi_x \sigma (1 - \beta) + \sigma \kappa (\psi_p - 1)}{(\sigma - \psi_x)^2 \beta^2}. \hspace{1cm} (48)$$

For (48) to be positive, since the denominator is positive, the numerator should be positive as well. This is the case if:

$$\kappa(\psi_p - 1) > -\psi_x(1 - \beta)^2 - \frac{\psi_x^2 (1 - \beta) \beta}{\sigma} - \frac{(\sigma - \psi_x) \beta \kappa}{\sigma}. \hspace{1cm} (49)$$

If conditions (47) and (49) are satisfied, then the system is determinate (Case II).

Now, if condition (49) is not satisfied, so that \( \mathcal{A}_4 < 0 \), we require that \( |\mathcal{A}_2| > 3 \) (Case III). Notice that

$$|\mathcal{A}_2| = \left| -\frac{(\sigma - \psi_x)(1 + \beta) + \sigma \beta + \kappa(1 - \psi_p)}{(\sigma - \psi_x)\beta} \right|. \hspace{1cm} (50)$$

Assume \( \psi_p \) is not very large, so \( \mathcal{A}_2 < 0 \). Then, condition \( |\mathcal{A}_2| > 3 \) requires that

$$-\frac{(\sigma - \psi_x)(1 + \beta) + \sigma \beta + \kappa(1 - \psi_p)}{(\sigma - \psi_x)\beta} < -3.$$  

or

$$\kappa(\psi_p - 1) < \sigma(1 - \beta) + \psi_x(2\beta - 1). \hspace{1cm} (51)$$

We show next that if condition (49) is not satisfied, then condition (51) is automatically satisfied. To this end, we need to check if, when

\[53\] If \( \psi_p \) is larger than one, condition (49) is always satisfied, so Case III becomes irrelevant.
\[ \kappa (\psi_p - 1) < -\psi_x (1 - \beta)^2 - \frac{\psi^2_x (1 - \beta) \beta}{\sigma} - \frac{(\sigma - \psi_x) \beta \kappa}{\sigma}, \]

it is always the case that condition (51) holds. This is the case if the right hand side of equation (51) is larger than the right hand side of equation (49). We prove this by contradiction. Suppose

\[ -\psi_x (1 - \beta)^2 - \frac{\psi^2_x (1 - \beta) \beta}{\sigma} - \frac{(\sigma - \psi_x) \beta \kappa}{\sigma} > \sigma (1 - \beta) + \psi_x (2 \beta - 1). \]

Rearranging,

\[ -\left( \psi_x (1 - \beta)^2 + \frac{\psi^2_x (1 - \beta) \beta}{\sigma} + \frac{(\sigma - \psi_x) \beta \kappa}{\sigma} + (\sigma - \psi_x) (1 - \beta) + \psi_x \beta \right) > 0, \]

which is clearly a contradiction because all terms inside the brackets in the left hand side of the previous expression are positive. We can conclude that all cases that satisfy \( \mathcal{P}(1) > 0 \) and \( \mathcal{P}(-1) < 0 \), either satisfy condition (49) or condition (51). This completes the proof.

### D.3 Determinacy of hybrid Wicksellian rules

The system composed by equations (1), (2) and (17) can be cast in the form:

\[ y_t = B E_t y_{t+1} + C z_t, \]

where \( y_t = [x_t; \pi_t; p_{t-1}] \), \( z_t = [r_t; u_t] \). In this particular case, the \( \tilde{B} \) matrix takes de form:

\[
\tilde{B} = \begin{pmatrix}
\frac{\sigma + \kappa (1 - \psi_p) + \psi_x}{\sigma \beta} & \frac{1 - \psi_x (1 + \beta)}{\sigma \beta} & \frac{\psi_x}{\sigma} \\
-\frac{\psi_x}{\beta} & \frac{1}{\beta} & 0 \\
0 & 1 & 1
\end{pmatrix}^{-1}. \tag{52}
\]

This system has two endogenous non-predetermined variables, \( x_t \) and \( \pi_t \), and one endogenous predetermined variable, \( p_{t-1} \). Thus, as in the case of the pure forward-looking Wicksellian rule analyzed previously, for the equilibrium to be determinate, that is, for the system to have a unique solution, it is required that the matrix \( B \) has two eigenvalues inside the unitary circle and one outside it.

To simplify the algebra, we define \( \tilde{B} = B^{-1} \), so to achieve determinacy, \( \tilde{B} \) needs to have two eigenvalues outside the unit circle.

The characteristic polynomial of matrix \( \tilde{B} \) is
\[ \mathcal{P}(\lambda) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0, \]

where \( \lambda \) are the eigenvalues of \( \tilde{B} \) and

\[
A_2 = \frac{\kappa(\psi_p - 1) - \psi_x \beta - \sigma}{\sigma \beta} - 2,
\]

\[
A_1 = \frac{\beta(\sigma + \psi_x) + \kappa + 2\sigma + \psi_x}{\sigma \beta},
\]

\[
A_0 = -\frac{\sigma + \psi_x}{\sigma \beta}.
\]

As a result, we can compute \( \mathcal{P}(1) \) and \( \mathcal{P}(-1) \) to obtain

\[
\mathcal{P}(1) = \frac{\kappa \psi_p}{\sigma \beta} > 0,
\]

(53)

\[
\mathcal{P}(-1) = \frac{\kappa(\psi_p - 2) - 4\sigma(1 + \beta) - 2\psi_x(1 + \beta)}{\sigma \beta}.
\]

(54)

Since \( \mathcal{P}(1) > 0 \) for all values of the parameters, as long as \( \psi_p > 0 \), we can already discard Case I as a relevant case to establish areas of determinacy. Instead, we consider Case II and Case III.

**D.3.1 Case II and III:**

These conditions require that \( \mathcal{P}(1) > 0 \) and \( \mathcal{P}(-1) < 0 \). The first condition is automatically satisfied. To satisfy the second condition, given that the denominator is always positive, the numerator must be negative, that is

\[
\kappa(\psi_p - 2) - 2(1 + \beta)\psi_x < 4\sigma(1 + \beta).
\]

(55)

Case II requires the following additional condition to hold

\[
\mathcal{A}_4 = \frac{\psi_x^2(1 - \beta) + \kappa \sigma (\psi_p + \beta - 1) - \psi_x (\kappa(\psi_p - 1) + \sigma(1 - \beta)^2)}{\sigma^2 \beta^2} > 0.
\]

(56)

For (56) to be positive, since the denominator is positive, the numerator should be positive as well. This is the case if
\[\psi_p > \frac{-\psi_x^2(1 - \beta) + \kappa \sigma (1 - \beta) + \psi_x \kappa - \psi_x \sigma (1 - \beta)^2}{\kappa (\sigma + \psi_x)}.\] \hspace{1cm} (57)

If conditions (55) and (57) are satisfied, then the system is determinate (Case II). If condition (57) is not satisfied, so that \(A_4 < 0\), we require \(|A_2| > 3\) (Case III). Notice that

\[|A_2| = \left|\frac{\kappa (\psi_p - 1) + \psi_x \beta - \sigma - 2 \sigma \beta}{\sigma \beta}\right|.

Assume \(\psi_p\) is not very large, so \(A_2 < 0\). Then, condition \(|A_2| > 3\) requires that

\[\frac{\kappa (\psi_p - 1) - \psi_x \beta - \sigma - 2 \sigma \beta}{\sigma \beta} < -3,
\]
or

\[\psi_p < \frac{\sigma (1 - \beta) + \psi_x \beta}{\kappa} + 1.\] \hspace{1cm} (58)

We show next that if condition (57) is not satisfied, then condition (58) is automatically satisfied. To this end, we need to check if, when

\[\psi_p < \frac{-\psi_x^2(1 - \beta) + \kappa \sigma (1 - \beta) + \psi_x \kappa - \psi_x \sigma (1 - \beta)^2}{\kappa (\sigma + \psi_x)},\]

it is always the case that condition (58) holds. This is the case if the right hand side of equation (58) is larger than the right hand side of equation (57). We prove this by contradiction. Suppose

\[\frac{-\psi_x^2(1 - \beta) + \kappa \sigma (1 - \beta) + \psi_x \kappa - \psi_x \sigma (1 - \beta)^2}{\kappa (\sigma + \psi_x)} > \frac{\sigma (1 - \beta) + \psi_x \beta}{\kappa} + 1.
\]

Rearranging,

\[\sigma^2(1 - \beta) + \sigma \psi_x \beta + \psi_x \sigma (1 - \beta) + \psi_x^2 + \sigma \kappa \beta + \psi_x \sigma (1 - \beta)^2 < 0,
\]

which is clearly a contradiction because all terms in the left hand side are positive. We can conclude that all cases that satisfy \(P(1) > 0\) and \(P(-1) < 0\), either satisfy condition (57) or condition (58). Then, the only relevant condition for determinacy is condition (55). This completes the proof.

\textsuperscript{54}If \(\psi_p\) is larger than one, condition (57) is always satisfied, so Case III becomes irrelevant.
D.4 Backward-looking rules

Here we prove Proposition 4. A backward-looking Taylor rule can be expressed as:

\[ i_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1}, \tag{59} \]

The system formed by equations (1), (2) and (59) can be rewritten as

\[ E_t y_{t+1} = \tilde{B} y_t + \tilde{C} z_t, \]

where \( y_t = [x_t; \pi_t; i_t] \), \( z_t = [r_t; u_t] \) and

\[ \tilde{B} = \begin{pmatrix}
\frac{\beta \sigma + \kappa}{\beta \sigma} & -\frac{1}{\beta \sigma} & \frac{1}{\sigma} \\
-\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\
\phi_x & \phi_\pi & 0
\end{pmatrix}. \tag{60} \]

This system has two endogenous non-predetermined variables, \( x_t \) and \( \pi_t \), and one endogenous predetermined variable, \( i_t \). We need to determine if matrix \( \tilde{B} \) defined in (60) has exactly two eigenvalues outside the unitary circle.

The characteristic polynomial of \( \tilde{B} \) is given by

\[ P(\lambda) = \lambda^3 + A_2 \lambda^2 + A_1 \lambda + A_0, \]

where \( \lambda \) are the eigenvalues of \( \tilde{B} \) and

\[ A_2 = \frac{-\beta \sigma + \kappa + \sigma}{\beta \sigma}, \]

\[ A_1 = \frac{-\beta \phi_x - \sigma}{\beta \sigma}, \]

\[ A_0 = \frac{\phi_\pi \kappa + \phi_x}{\beta \sigma}. \]

We can compute \( P(1) \) and \( P(-1) \) to obtain

\[ P(1) = \frac{\kappa (\phi_\pi - 1) + \phi_x (1 - \beta)}{\beta \sigma}, \]

\[ P(-1) = \frac{-2 \beta \sigma - \kappa (1 - \phi_\pi) - 2 \sigma + \phi_x (1 + \beta)}{\beta \sigma}. \]

We consider cases I and II separately.
D.4.1 Case I:

Notice that, in this case, $\mathcal{P}(1) < 0$ and $\mathcal{P}(-1) > 0$ if the following conditions are satisfied:

$$\kappa(\phi - 1) + \phi_x(1 - \beta) < 0,$$

$$\kappa(\phi - 1) + \phi_x(1 + \beta) > 2\sigma(1 + \beta).$$

D.4.2 Case II:

Notice that, in this case, $\mathcal{P}(1) > 0$ and $\mathcal{P}(-1) < 0$ if the following conditions are satisfied:

$$\kappa(\phi - 1) + \phi_x(1 - \beta) > 0,$$

$$\kappa(\phi - 1) + \phi_x(1 + \beta) < 2\sigma(1 + \beta).$$

Moreover, we need that

$$A_4 = \frac{(\phi_x + \phi_x)(\phi_x + \phi_x + \beta\sigma + \kappa + 1) - \beta^2\sigma\phi_x + \sigma^2\beta - \beta^2\sigma^2}{\beta^2\sigma^2} > 0.$$

For this expression to be positive, it is required that the numerator is positive. Rearranging the numerator, it can be expressed as

$$\phi_x\kappa(\phi_x + \phi_x + \beta\sigma + \kappa + \sigma) + \phi_x(\phi_x + \phi_x + \phi_x + \kappa + \sigma) + \phi_x\beta\sigma - \beta\sigma^2(\beta - 1),$$

which is clearly positive because $\beta < 1$ and, thus, all terms in the previous expression are positive. This completes the proof.

Now we proceed to provide a heuristic proof of Claim 1. A backward-looking Wicksellian rule reads:

$$i_t = \psi_p(p_{t-1} - \bar{p}) + \psi_x x_{t-1}, \quad \text{(61)}$$

As before, the system composed by equations (1), (2) and (61) can be cast in the form:

$$E_t y_{t+1} = \tilde{B}y_t + \tilde{C}z_t + \tilde{D}\bar{p},$$

where $y_t = [x_t; \pi_t; i_t; p_{t-1}]$, $z_t = [r^x_t; u_t]$. In this particular case, the $\tilde{B}$ matrix takes de form:
\[
\tilde{B} = \begin{pmatrix} 
\frac{\beta \sigma + \kappa}{\beta \sigma} & -\frac{1}{\beta \sigma} & \frac{1}{\sigma} & 0 \\
-\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 & 0 \\
\psi_x & \psi_p & 0 & \psi_p \\
0 & 1 & 0 & 1
\end{pmatrix}.
(62)
\]

Notice that this system has two endogenous non-predetermined variables, \(x_t\) and \(\pi_t\), and two endogenous predetermined variables, \(i_t\) and \(p_{t-1}\). In this case, for the equilibrium to be determinate it is required that the matrix \(\tilde{B}\) has two eigenvalues outside the unitary circle and two inside.

Given that matrix \(\tilde{B}\) is a \(4 \times 4\) matrix, we cannot characterize analytically the regions of determinacy, indeterminacy and explosive solutions. Instead, we claim that the region of determinacy is the same as the one for forward-looking and hybrid Wicksellian rules (see Claim 1).

Notice that, if Claim 1 is correct, backward-looking Wicksellian rules do not need to satisfy the Taylor principle. In this case, however, we cannot ascertain that a Wicksellian rule leads to a larger area of determinacy than a Taylor rule, because there is an additional area of determinacy in the Taylor rule for which the Taylor principle is not satisfied.

We assess the validity of Claim 1 through a numerical example in the spirit of the ones conducted before. The darker area corresponds to combinations of parameters \(\{\phi_x, \phi_x\}\) of the Taylor rule and parameters \(\{\psi_x, \psi_p\}\) of the Wicksellian rule for which the systems are determinate. The semi-dark shaded area corresponds to combinations of parameters \(\{\phi_x, \phi_x\}\) of the Taylor rule for which the system is determinate, but to parameters \(\{\psi_x, \psi_p\}\) of the Wicksellian rule for which the system is explosive. The lighter shaded area corresponds to combinations of parameters \(\{\phi_x, \phi_x\}\) of the Taylor rule for which the system is indeterminate, but to parameters \(\{\psi_x, \psi_p\}\) of the Wicksellian rule for which the system is determinate. Finally, the non-shaded area corresponds to combinations of parameters \(\{\phi_x, \phi_x\}\) of the Taylor rule and parameters \(\{\psi_x, \psi_p\}\) of the Wicksellian rule for which the systems are explosive.

As it is clear from Figure 9, the region of determinacy of a Wicksellian backward-looking rule is identical to the region of determinacy under a forward-looking rule. Then, Claim is confirmed (in this particular example). From the numerical example, it is not possible to conclude that one area of determinacy is larger than the other. It is still the case, however, that the Wicksellian rule does not need to satisfy the Taylor principle. The Taylor rule, on

\footnote{To our knowledge, in the case of a \(4 \times 4\) matrix there are no theorems that characterize regions where the eigenvalues are inside or outside the unitary circle, as is the case with \(3 \times 3\) matrices (see Woodford (2003)).}
the contrary, has to satisfy this principle for some combinations of parameters \( \{ \phi_x, \phi_x \} \) in order to attain determinacy.

D.5 Systematic Bias in Simple Taylor Rules

Here we prove Proposition 3. The Wicksellian rule in (22) is equivalent to a simple Taylor rule of the form:

\[ i_t = i_{t-1} + \psi_p \pi_t. \] (63)

As a consequence, the OLS estimator of \( \phi_\pi \) in (21) is a biased estimator of \( \psi_p \). The bias is determined by the correlation between the explanatory variable in (21), \( \pi_t \), and the omitted variable, \( i_{t-1} \). In particular,

\[ E(\hat{\phi}_\pi) = \psi_p + E \left( \frac{\sum_{t=0}^T \pi_{t-1}}{\sum_{t=0}^T \pi_t^2} \right), \] (64)

where \( E = \left( \frac{\sum_{t=0}^T \pi_{t-1}}{\sum_{t=0}^T \pi_t^2} \right) \) is the bias. Under the Wicksellian rule prices are stationary and determined according to equation (20), \( p_t = a_p r_t^e + b_p u_t + c_p p_{t-1} \), where \( 0 < c_p < 1 \). As a consequence, it can be shown that:

\[ E(\pi_t^2) = \frac{2 \left[ (c_p - 1)b_p E(p_{t-1}u_t) + (c_p - 1)a_p E(p_{t-1}r_t^e) + b_p^2 E(u_t^2) + a_p^2 E(r_t^e)^2 + 2a_bp E(u_t r_t^e) \right]}{(1 + c_p)}. \] (65)

Under the Wicksellian rule (22) it is possible to derive the following expression for the lagged interest rate, \( i_{t-1} = \psi_p (\pi_t + \pi_{t-1} + \ldots + \pi_{t-\tau+1}) \), where \( p_{t-\tau} \) converges to zero as \( \tau \) increases. Based on the previous expression, it is possible to express \( \pi_t i_{t-1} \) as:

\[ E(\pi_t i_{t-1}) = \psi_p \left[ \pi_t \pi_{t-1} + \pi_t \pi_{t-2} + \ldots + \pi_t \pi_{t-1+\tau} \right] \\
= \psi_p \left[ (c_p - 1)(1 - c_p) E(p_t^2)(1 + c_p + c_p^2 + \ldots + c_p^{\tau-1}) + \\
+ b_p E(p_{t-1}u_t) + a_p E(p_{t-1}r_t^e) + E(k_t) + E(z_t) \right] \] (66)

where \( k_t = -p_{t-\tau} \left[ (c_p - 1)b_p \sum_{j=1}^{\tau-1} c_p^{j-1} u_{t-j} + b_p u_t \right] \) and \( z_t = -p_{t-\tau} \left[ (c_p - 1)a_p \sum_{j=1}^{\tau-1} c_p^{j-1} r_{t-j}^e + a_p r_t^e \right] \).
and we have used the fact that $E(p_i^2) = E(p^2)$ for any $t$. Notice that both $k_t$ and $z_t$ converge to zero as $\tau$ increases. Hence, based on (65) and (66), we conclude that:

$$E \left( \hat{\theta}_\pi \right) = \frac{1}{2} \psi_p + E \left( \frac{\sum_{t=0}^{T}(k_t + z_t)}{\sum_{t=0}^{T} \pi_t^2} \right)$$

where, for a large $\tau$, $k_t$ and $z_t$ converge to zero for any $t = 0$ to $T$. This concludes the proof.
References


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